Introduction to modeling using bond graphs

Basic components

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October 22, 2014

In this lecture we will proceed further into the popular modeling methodology based on *bond graphs*. This technique generalizes the concept of n-ports well known from the electrical engineering—by connecting two components or subsystems the interaction between the two is realized by an exchange of energy. The rate of transfer of energy (the power) is given as a product of two variables—voltage and current in electrical circuits, force and velocity in mechanical translation and so on. We will now develop a set of basic types of general(ized) components—generalized sources, generalized loads, generalized resistors, generalized transformers, generalized gyrators, generalized inertances and generalized compliances. Only after introducing these will we be prepared for building more complex models.

1 Half-arrow notation in bond graphs

Although the key information about the existence of a power bond between two subsystem or components has already been included in the bond graph, there is still one more useful information that needs to be added. This information is graphically encoded as a half-arrow as in Fig. 1. It is vital to understand the meaning of this new symbol correctly! It shows the direction of flow of energy if both e and $\dot{q}$ have the same sign, that is, when $\mathcal{P} > 0$; it does not suggest that the energy is flowing exclusively from A to B. The energy can easily flow from B to A.

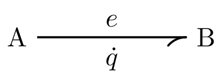


Figure 1: Simple bond graph depicting an energy exchange (power bond) between A and B, now including also the half-arrow, which shows the direction of flow of energy if both e and $\dot{q}$ are positive.

As an example, let us again consider the tug-of-war game as in Fig. 2

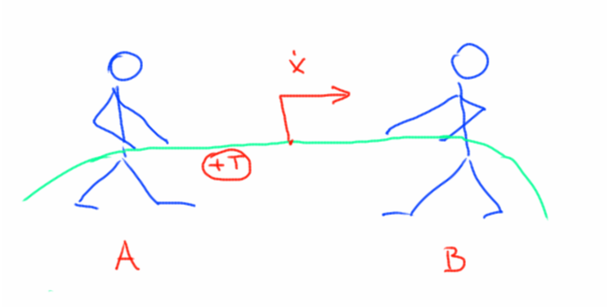
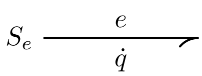


Figure 2: Sketch of a tug-of-war game.

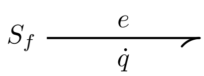
As we have already agreed, the (generalized) force is a scalar variable here, we only need to agree that it is positive in extension (well, we can hardly experience pushing while using a rope). Say, the team B is dominating, that is, moving backward whereas the team A is yielding, that is, moving forward. What is the direction of the half-arrow? In which direction does the energy flow if the team A starts dominating, that is, moving backwards? Will then the direction of the half-arrow change?

2 Sources and sinks

2.1 Ideal sources and sinks

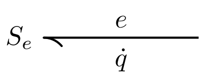


(a) Ideal source of generalized effort (or force).

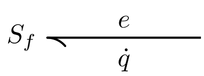


(b) Ideal source of generalized flow (or velocity).

Figure 3: Ideal sources.



(a) Ideal sink of generalized effort (or force).



(b) Ideal sink of generalized flow (or velocity).

Figure 4: Ideal sinks.

2.1.1 Examples

- mechanical: gravitational force

- electrical: ideal generator of electrical voltage or current
- hydraulic

2.2 Nonideal sources

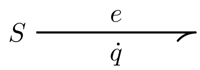


Figure 5: Nonideal source.

Generally nonlinear load characterictics $e = e(\dot{q})$ or $\dot{q} = \dot{q}(e)$.

2.3 Generalized resistors

Although one symbol (S) will be enough for both sources and sinks (the distinction between them done via the half-arrow), we still introduce a new symbol for generalized sinks—R—and will call them *generalized resistors.*

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Resistor relates e and $\dot{q}$. Nonlinear resistor can be given by

$$e = e(\dot{q}) \tag{1}$$

or by

$$\dot{q} = \dot{q}(e). \tag{2}$$

A linear resistor is given by its *modulus*

$$\boxed{e = R\dot{q}} \tag{3}$$

or by

$$\boxed{\dot{q} = \frac{e}{R}.} \tag{4}$$

2.3.1 Examples

- mechanical: friction
- electrical: resistor
- hydraulic: viscous friction

3 Ideal machines—generalized transformers and gyrators

3.1 Generalized transformers

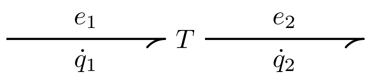

Figure 6: Generalized transformer.

$$\boxed{\dot{q}_2(t) = T\dot{q}_1(t)} \tag{5}$$

3.1.1 Examples

- mechanical: gears
- electrical: ideal transformer
- hydraulic:
- from one domain to another—transducers: pump

3.2 Generalized gyrators

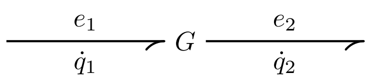

Figure 7: Generalized gyrator.

$$\boxed{e_2(t) = G\dot{q}_1(t)} \tag{6}$$

3.2.1 Examples

- mechanical: a gyroscope
- electrical: a circuit built from passive and active components
- hydraulic: ?
- from one domain to another—transducers: electrical DC motor with a permanent magnet

4 Ideal accumulators of energy—generalized compliances and inertances

4.1 Generalized compliances

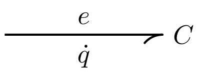

Figure 8: Generalized compliance.

$$\boxed{e(t) = \frac{1}{C} q(t)} \tag{7}$$

Accumulate energy proportional to *generalized displacement* q

$$\mathcal{V} = \frac{1}{2} \frac{1}{C} q^2(t) = \frac{1}{2} C e^2(t) \tag{8}$$

4.1.1 Examples

- mechanical: spring, torsional spring
- electrical: capacitor
- hydraulic: reservoir

4.2 Generalized inertances

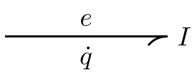

Figure 9: Generalized inertance.

New variable introduced—generalized momentum $p(t)$—and it is defined as

$$p(t) = \int e(\tau) \mathrm{d}\tau \tag{9}$$

or

$$e(t) = \dot{p}(t). \tag{10}$$

The component called *generalized inertance* is then defined as

$$\boxed{p(t) = I \dot{q}(t)} \tag{11}$$

It accumulates an energy proportional to *generalized velocity* $\dot{q}$.

$$\mathcal{T} = \frac{1}{2} I \dot{q}^2(t) = \frac{1}{2}\frac{1}{I} p^2(t) \tag{12}$$

4.2.1 Examples

- mechanical: mass traveling at some velocity, mass rotating around an axis
- electrical: inductor
- hydraulic: volume of an incompressible liquid flowing at some (volumetric) flow rate.

5 Summary

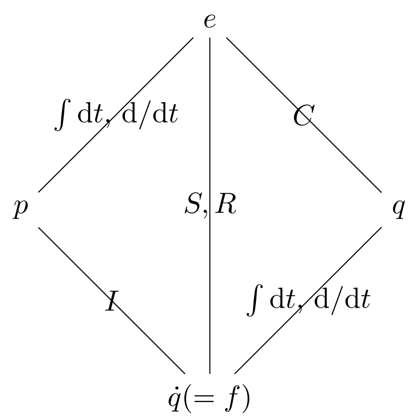

Figure 10: Relationship among the important (generalized) variables. Note that the horizontal connection between p and q has been recently (2008) invented—the memristor element.

6 Literature

The lecture was prepared mainly using the sections 2.1 through 3.2 in [2]. There were a few important issues that we barely scratched during the lecture and you were asked to study these on your own. In particular, the topic of cascading the transformers and gyrators and the topic of matching the load to a source using a transformer, which are covered by 2.5.

As a complementary text you can use the popular [5]. However, this book is probably not available in many copies in the library, if any.

The tutorial paper [4] can serve its purpose as well. You can get it from the *IEEE Xplore* library within the institutional subscription. Another tutorial paper is [1], which can be found online.

The book [3], now available online, can also be used. However, the quality of the scanned version is poor.

References

[1] J.F. Broenink. Introduction to physical systems modelling with bond graphs. *SiE Whitebook on Simulation Methodologies*, pages 1–31, 1999.

[2] Forbes T. Brown. *Engineering System Dynamics: A Unified Graph-Centered Approach, Second Edition*. CRC Press, 2nd edition, August 2006.

[3] François E. Cellier and Jurgen Greifeneder. *Continuous System Modeling*. Springer, 1st edition, May 1991.

[4] P. J Gawthrop and G. P Bevan. Bond-graph modeling. *IEEE Control Systems*, 27(2):24–45, April 2007.

[5] Dean Karnopp, Donald L Margolis, and Ronald C Rosenberg. *System dynamics modeling, simulation, and control of mechatronic systems*. John Wiley & Sons, Hoboken, N.J., 5th edition, 2012.