Simple bond graphs

Interconnection, causality

Zdeněk Hurák

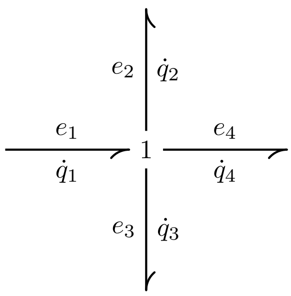
October 16, 2018

We have previously introduced basic general(ized) elements for bond graphs. In this lecture we are going to start interconnecting them in order to obtain models of systems. Although some more advanced elements and techniques for bond graphs are still to be studied, we are now perfectly ready to use the tools we already have to build mathematical models of simple yet realistic systems.

1 Junctions

In order to connect more then two elements, we use *junctions*. Ideally these do not store, do not produce, do not dissipate energy. There are two variants of junctions in bond graphs.

1.1 Type-1 junctions



Type-1 junction.

Also *common-flow* or *common-velocity* junction

$$\boxed{\dot{q}_1 = \dot{q}_2 = \dot{q}_3 = \dot{q}_4.} \tag{1}$$

The conservation of energy gives

$$\underbrace{e_1\dot{q}_1}_{\mathcal{P}_1} - \underbrace{e_2\dot{q}_2}_{\mathcal{P}_2} - \underbrace{e_3\dot{q}_3}_{\mathcal{P}_3} - \underbrace{e_4\dot{q}_4}_{\mathcal{P}_4} = 0, \tag{2}$$

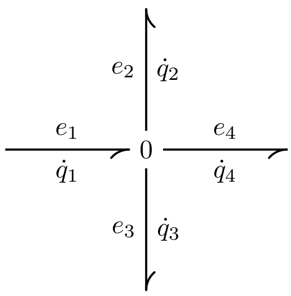
from which it follows

$$e_1 + e_2 + e_3 - e_4 = 0. \tag{3}$$

1.1.1 Examples

- mechanical: a single velocity and several forces
- electrical: common current, hence series interconnection
- hydraulic: common volumetric flow: pump—pipe—valve—hydraulic motor—nozzle—reservoir.

1.2 Type-0 junctions



Type-0 junction.

Also *common-effort* junction

$$\boxed{e_1 = e_2 = e_3 = e_4.} \tag{4}$$

The conservation of energy holds identically as in the type-1 case

$$\underbrace{e_1\dot{q}_1}_{\mathcal{P}_1} - \underbrace{e_2\dot{q}_2}_{\mathcal{P}_2} - \underbrace{e_3\dot{q}_3}_{\mathcal{P}_3} - \underbrace{e_4\dot{q}_4}_{\mathcal{P}_4} = 0, \tag{5}$$

from which it follows

$$\dot{q}_1 + \dot{q}_2 + \dot{q}_3 - \dot{q}_4 = 0. \tag{6}$$

1.2.1 Examples

- mechanical: single force, several velocities
- electrical: common node, hence parallel interconnection
- hydraulic: single pressure, several nozzles.

2 Simple systems

2.1 Electrical

- Identify all the distinct voltages and currents in the circuit including the directions (using arrows) for currents and polarities (using + and − symbols) for voltages (or voltage drops).
- Start drawing the bond graph by putting down the type-0 junctions corresponding to the *nodes* in the circuit, that is, the locations with a distinct voltage (or actually a potential).
- Include also the ground node. The associated bond will be erased later the related bonds because the voltage (generalized force) is zero, hence the power is zero too. But put it there just to make the modeling procedure more transparent.
- Use the type-1 junctions to insert R, C and I elements that depend on differences between the nodal voltages (or actually potentials). Similarly, add the S elements (sources) between the two type-0 junctions.

2.2 Mechanical

- Identify all the distinct velocities in the system and highlight them in the technical drawing using arrows. Identify also all the forces. For springs and dampers, choose a polarity of the force: type either +C or +T next to the element, if the force is positive in compression or tension, respectively.
- Start drawing the bond graph by putting down the type-1 junctions corresponding to the velocities identified in the previous step.
- Include also the velocity of the reference frame. The associated bond will be erased later because the velocity is zero, hence the power is zero. But please put it there just to make the modeling procedure more transparent. After all, what is regarded as fixed now can be attached to some moving carrier in some future extension.

- Use type-0 junctions to insert R and C elements that depend on the relative velocities. Attach the I elements directly to the type-1 junctions. Note that this is where the electric and mechanical worlds are fundamentally different. The mechanical instance of a generalized inertance accumulates energy proportionally to the absolute velocity. There is no point in considering relative velocity when considering a kinetic energy!
- Type-0 junctions can also be used together with transformers to include *kinematic constraints*.

2.3 Hydraulic

Pretty much following the strategy for the electric systems.

- Label the spots with the distinct pressures. Note the caveat, though. Unlike in low-frequency electric circuits, where we consider all energy-dissipation and energy-accumulation effects concentrated in dimensionless spots (that is also why they are called *lumped systems*), in hydraulic systems such simplifying assumption is hardly possible. For example, the pressure is continuously decreasing along the pipe due to energy dissipation. Nonetheless, we at least identify the key points in the system: at the inlet, at the outlet, before the valve, after the valve, at the bottom of the reservoir and so on. Among these choose those with a distinc pressure and label it.
- Similarly as in electrical systems, start drawing the bond graph by putting down the type-0 junctions that correspond to the distinct pressures identified above.
- Include also the ambient pressure. Note that it has the same role as the ground in electric circuits. However, you cannot neglect it unless you switch from using the absolute pressure to the gauge pressure. It is only in this particular case that the corresponding bonds could be erased because the gauge pressure for the atmosphere will be zero.
- Use type-1 junctions to insert relative elements such as R, C and I. But beware that unlike in the electric circuits, here the I and R elements (corresponding to an inertia of the moving fluid and a viscous drag in the pipe) are attached to the same type-1 junction. This is one of the artefacts of the fact that we are approximating the inherently spatially distributed system (also called distributed-parameter system) with lumped elements.

3 Literature

This lecture was prepared using mainly the sections 3.3 through 3.5 in [2]. Some more modeling strategies and examples in sections 5.1 through 5.3 of the same book. (Note that we can skip safely the chapter 4 altogether as it introduces some fundamental techniques for analysis of linear models to which you have already been exposed).

Complementary reading is in chapters 3 through 5 in the popular [4]. This book is indeed a nice complementary reading. Consider getting a copy.

Similarly as in the previous lecture which introduced the very basic concepts and compoments for bond graphs, you can consult the tutorial paper [3]. Another tutorial paper available online is [1], which can be found online.

References

[1] J.F. Broenink. Introduction to physical systems modelling with bond graphs. *SiE Whitebook on Simulation Methodologies*, pages 1–31, 1999.

[2] Forbes T. Brown. *Engineering System Dynamics: A Unified Graph-Centered Approach, Second Edition*. CRC Press, 2nd edition, August 2006.

[3] P. J Gawthrop and G. P Bevan. Bond-graph modeling. *IEEE Control Systems*, 27(2):24–45, April 2007.

[4] Dean Karnopp, Donald L Margolis, and Ronald C Rosenberg. *System dynamics modeling, simulation, and control of mechatronic systems*. John Wiley & Sons, Hoboken, N.J., 5th edition, 2012.