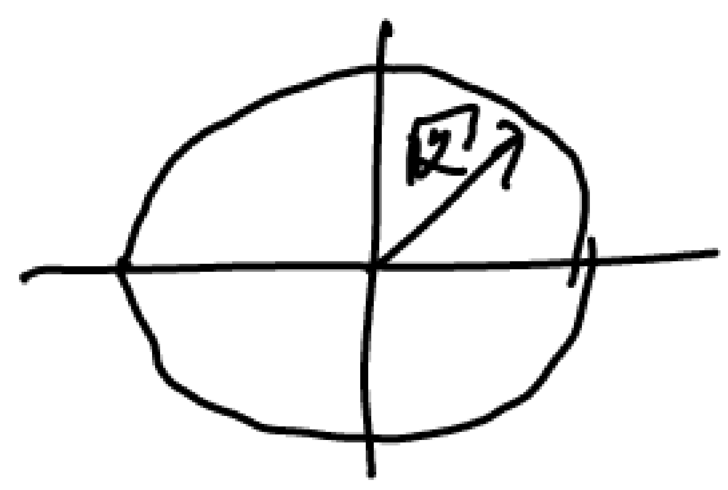
VARIANTA B

① a) $x^2+y^2-1>0$

Definiční obor je $D_f=\{(x,y)\in\mathbb{R}^2 \mid x^2+y^2>1\}$

$f(x,y)=0 \ldots \; x^2+y^2-1=1$. Tedy $x^2+y^2=2$.

Proto $\mathrm{ho}(f;0)=\{(x,y)\in\mathbb{R}^2 \mid x^2+y^2=2\}$



b) $\dfrac{\partial f}{\partial x}(-1,1)=\dfrac{2x}{x^2+y^2-1}\Bigg|_{\substack{x=-1\\y=1}}=-2$

$\dfrac{\partial f}{\partial y}(-1,1)=\dfrac{2y}{x^2+y^2-1}\Bigg|_{\substack{x=-1\\y=1}}=2$

$\Rightarrow g'(3)=\nabla f(-1,1)\cdot\varphi'(3)=(-2,2)\cdot(1,3)=4$

② $4x^3-2y=0 \quad (1)$

$-2x+2y=0 \quad (2)$

Z (2) máme $x=y$. Dosazením do (1) dostaneme $2x(2x^2-1)=0$.

Tedy $x=0$ nebo $x=\pm\frac{1}{\sqrt{2}}$.

Stacionární body jsou $(0,0)$, $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ a $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$.

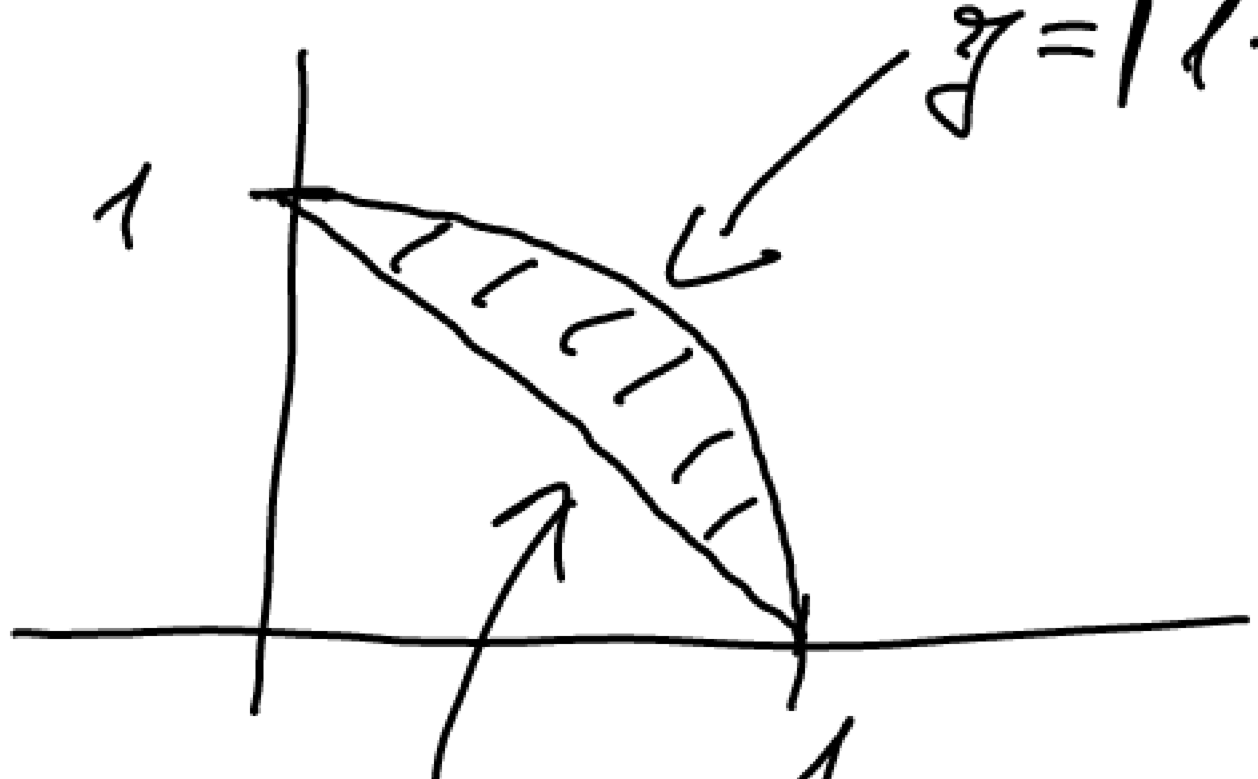
$$H_f(x,y) = \begin{pmatrix} 12x^2 & -2 \\ -2 & 2 \end{pmatrix}$$

$$H_f(0,0) = \begin{pmatrix} 0 & -2 \\ -2 & 2 \end{pmatrix}$$ je indefinitní $\Rightarrow (0,0)$ je sedl. bod.

$$H_f\left(\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right) = \begin{pmatrix} 6 & -2 \\ 2 & 2 \end{pmatrix}$$ je pozit. def. $\Rightarrow \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ jsou body lok. min.

③

$y = \sqrt{1-x^2}$, tj. $x^2+y^2=1$



$y = 1-x$

$y = -x+1$ … d. m.

$y = \sqrt{1-x^2}$ … h. m.

$y = 1 - x$ … $r\sin\varphi = 1 - r\cos\varphi \Rightarrow r = \frac{1}{\sin\varphi + \cos\varphi}$

Proto

$$\int_0^1 \int_{1-x}^{\sqrt{1-x^2}} \frac{x+y}{x^2+y^2}\, dy\, dx = \int_0^{\frac{\pi}{2}} \int_{\frac{1}{\sin\varphi+\cos\varphi}}^{1} \frac{r(\cos\varphi + \sin\varphi)}{r^2}\, r\, dr\, d\varphi$$

$$= \int_0^{\frac{\pi}{2}} \cos\varphi + \sin\varphi - 1\, d\varphi = \Big[\sin\varphi - \cos\varphi - \varphi\Big]_0^{\frac{\pi}{2}} = 2 - \frac{\pi}{2}.$$

④ $$f(x)=\frac{1}{2(x-3)+5}=\frac{1}{5}\,\frac{1}{1-\left[-\frac{2}{5}(x-3)\right]}$$

$$=\frac{1}{5}\sum_{k=0}^{\infty}\frac{(-1)^k 2^k}{5^k}(x-3)^k=\sum_{k=0}^{\infty}\frac{(-1)^k 2^k}{5^{k+1}}(x-3)^k$$

για $\left|-\frac{2}{5}(x-3)\right|<1 \iff |x-3|<\frac{5}{2}$ (δηλ: $R=\frac{5}{2}$).