VARIANTA C

1 a) $\nabla f(4,4) = \left(\frac{1}{2\sqrt{x}} e^{x-y} + \sqrt{x} e^{x-y}, -\sqrt{x} e^{x-y} \right) \Big|_{\substack{x=4 \\ y=4}}$

$= \left(\frac{9}{4}, -2 \right)$

b)

$T_1(x,y) = f(4,4) + \nabla f(4,4) \cdot (x-4, y-4)$

$= 2 + \frac{9}{4}(x-4) - 2(y-4)$

$f\left(\frac{41}{10}, \frac{41}{10}\right) \approx T_1\left(\frac{41}{10}, \frac{41}{10}\right) = 2 + \frac{9}{40} - \frac{8}{40} = 2 + \frac{1}{40}$

2 Hledáme maximum funkce $f(x,y) = xy$ na množině $M = \{(x,y) \in \mathbb{R}^2 \mid x^2 + 3y^2 = 1, x \geq 0, y \geq 0\}$

$y + \lambda 2x = 0 \quad (1)$

$x + \lambda 6y = 0 \quad (2)$

$x^2 + 3y^2 = 1 \quad (3)$

$x(1) - y(2) \ \dots \ 2\lambda(x^2 - 3y^2) = 0$

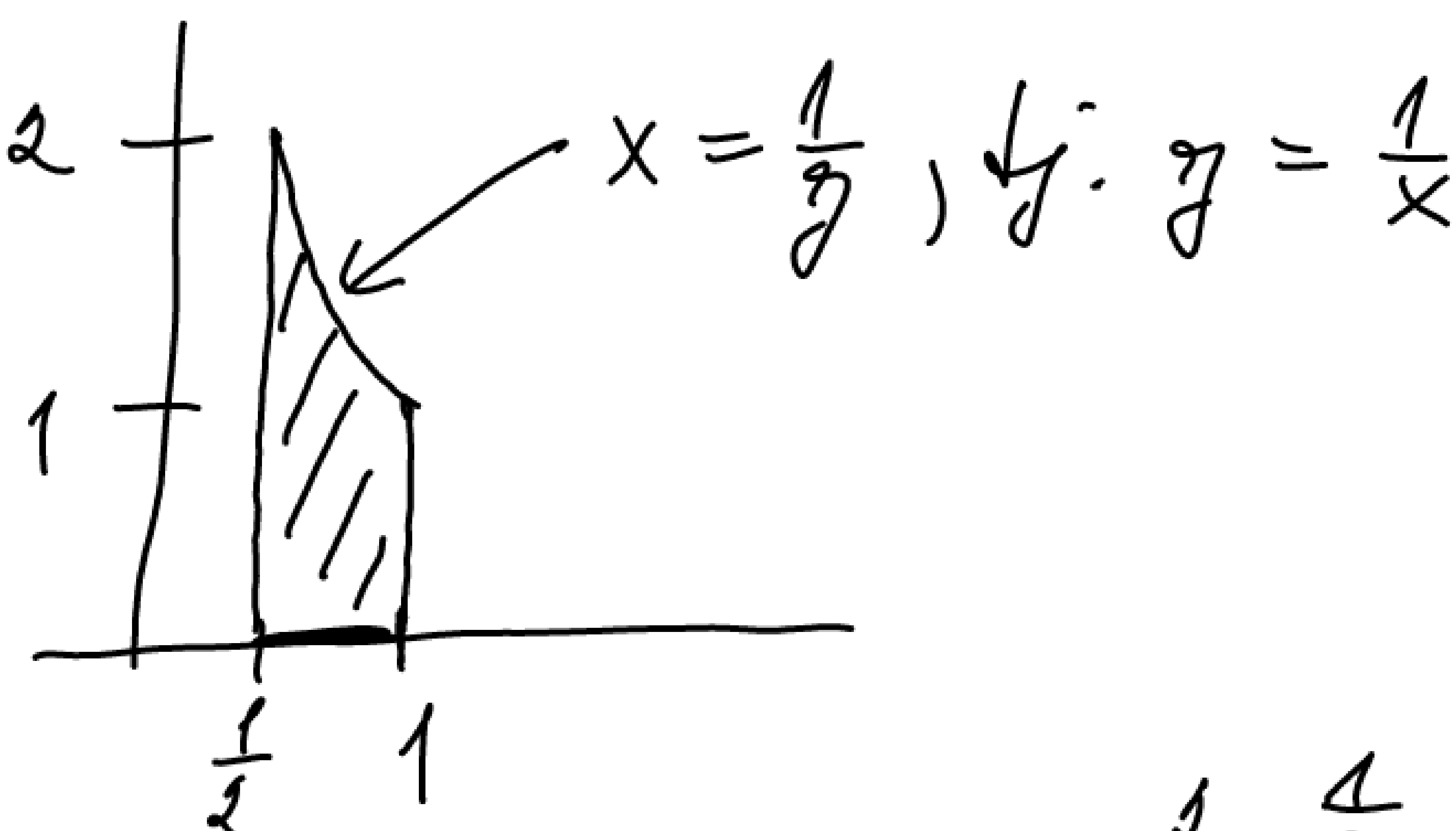
- $\lambda = 0 \ \dots \ x = y = 0$ spor s (3).
- $\lambda \neq 0 \ \dots \ x^2 = 3y^2$. Dosazením do (3)

máme $2x^2 = 1$. Tedy $x = \pm \frac{1}{\sqrt{2}} \overset{x \geq 0}{\Longrightarrow} x = \frac{1}{\sqrt{2}}$

Odtud $y^2 = \frac{1}{6} \overset{y \geq 0}{\Longrightarrow} y = \frac{1}{\sqrt{6}}$

Podezřelé body z maxima $(1,0)$, $\left(0,\frac{1}{\sqrt{3}}\right)$, $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{6}}\right)$. Protože $f(1,0)=f\left(0,\frac{1}{\sqrt{3}}\right)=0<f\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{6}}\right)=\frac{1}{2\sqrt{3}}$, je $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{6}}\right)$ hledaný bod maxima.

(3)



$$\int_0^1\int_{\frac12}^1 x\sin(\pi x)\,dx\,dy+\int_1^2\int_{\frac12}^{\frac1y} x\sin(\pi x)\,dx\,dy$$

$$=\int_{\frac12}^1\int_0^{\frac1x} x\sin(\pi x)\,dy\,dx=\int_{\frac12}^1 \sin(\pi x)\,dx=\left[-\frac{\cos(\pi x)}{\pi}\right]_{\frac12}^1$$

$$=\frac{1}{\pi}$$

(4) $$\sum_{k=0}^{+\infty}(-3)^k(x+1)^{k+1}=(x+1)\sum_{k=0}^{\infty}\left(-3(x+1)\right)^k$$

$$=\frac{x+1}{1+3(x+1)}=\frac{x+1}{3x+4}\quad \text{pro } |x+1|<\frac13$$

(tj. $R=\frac13$)