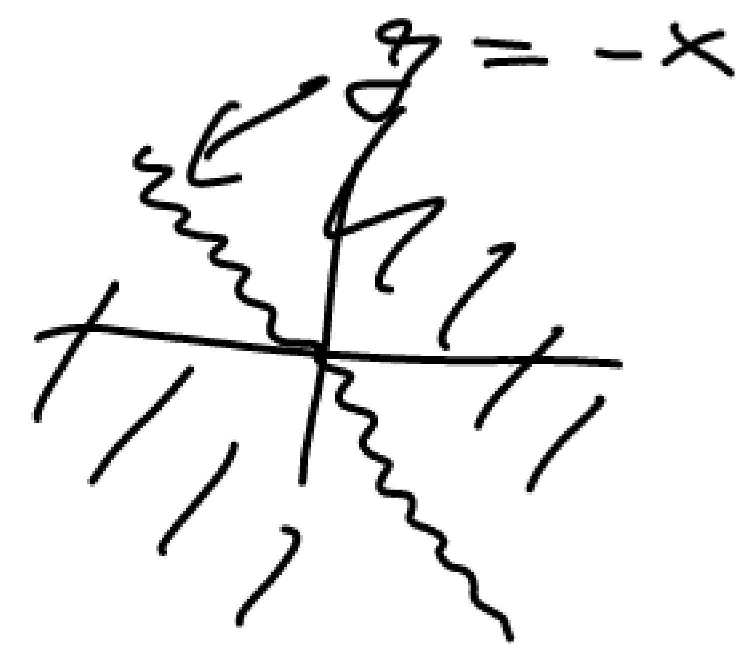
VARIANTA D



① a) $D_f = \{(x,y)\in\mathbb{R}^2 \mid y \neq -x\}$

$$\frac{x-y^2}{x+y}=1 \quad \dots \quad x-y^2 = x+y$$

$$y(y+1)=0 \iff y=0 \text{ nebo } y=-1$$

$$\mathrm{lev}(f;1) = \{(x,y)\in\mathbb{R}^2 \mid (y=0 \text{ a } x\neq 0) \text{ nebo } (y=-1 \text{ a } x\neq -1)\}$$

b) $\frac{\partial f}{\partial x} = \frac{x+y-(x-y^2)}{(x+y)^2} = \frac{y+y^2}{(x+y)^2}$

$$\frac{\partial f}{\partial y} = \frac{-2y(x+y)-(x-y^2)}{(x+y)^2} = \frac{-2yx-y^2-x}{(x+y)^2}$$

$$\Rightarrow g'(0) = Df(-2,1)\bullet\varphi'(0) = (2,5)\bullet(-5,2) = 0$$

② $y^2-2x+2y-3=0 \quad (1)$

$2xy+2x=0 \quad (2)$

$(2) \Rightarrow x=0$ nebo $y=-1$

- $x=0 \overset{(1)}{\Rightarrow} y^2+2y-3=0$, tj. $(y+3)(y-1)=0$

  $y=-3$ nebo $y=1$

- $y=-1 \overset{(1)}{\Rightarrow} x=-2$

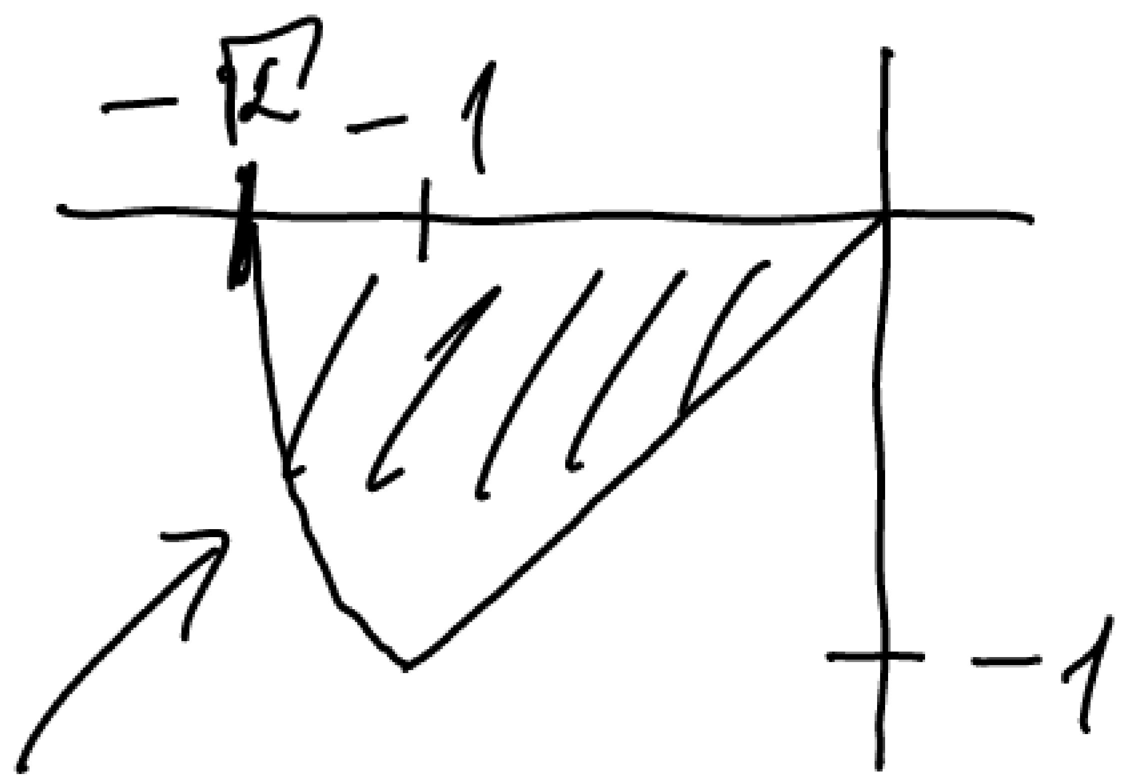
$$H_f(x,y) = \begin{pmatrix} -2 & 2+2y \\ 2+2y & 2x \end{pmatrix}$$

$$H_f(0,-3) = \begin{pmatrix} -2 & -4 \\ -4 & 0 \end{pmatrix} \dots \text{indef.} \Rightarrow (0,-3) \text{ je sedl. bod.}$$

$$H_f(0,1) = \begin{pmatrix} -2 & 4 \\ 4 & 0 \end{pmatrix} \dots \text{— " —} \Rightarrow (0,1) \text{ — " —}$$

$$H_f(-2,-1) = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \dots \text{negat. def.} \Rightarrow (-2,-1) \text{ je bod lok. max.}$$

(3)



$x = y$

$x = -\sqrt{2-y^2}$

$y \in [0,1]$

$x = -\sqrt{2-y^2}$, tj. $x^2+y^2=2$

$$\int_{-1}^{0}\int_{-\sqrt{2-y^2}}^{y} y \, dx\, dy = \int_{-\pi}^{-\frac{3\pi}{4}}\int_{0}^{\sqrt{2}} r\sin\varphi \, r \, dr\, d\varphi = \frac{2\sqrt{2}}{3}\Big[-\cos\varphi\Big]_{-\pi}^{-\frac{3\pi}{4}}$$

$$= -\frac{2\sqrt{2}}{3}\left(-\frac{\sqrt{2}}{2}+1\right) = \frac{2}{3}\left(1-\sqrt{2}\right)$$

(4) $$f(x) = \frac{2}{2-(x+1)} = \frac{1}{1-\frac{x+1}{2}} = \sum_{k=0}^{\infty} \frac{1}{2^k}(x+1)^k$$

pro $|x+1| < 2$ (tj. $R = 2$).