INTRODUCTION TO MPC — TRACKING

$y_k \longrightarrow r_k \implies e_k := r_k - y_k \longrightarrow 0 \implies \boxed{e_k := r_k - C x_k \to 0}$

output, reference

$y_k = C x_k + \cancel{D u_k}$

no feedthrough

@ time t:

$$\min \; \frac{1}{2} e_{t+N}^T S e_{t+N} + \frac{1}{2} \sum_{k=0}^{N-1} \left(e_{t+k}^T Q e_{t+k} + u_{t+k}^T R u_{t+k} \right)$$

!!!

in steady state $e_k = 0$, $u_k \neq 0$

$\implies$ no steady state

Solution: $\boxed{\text{USE } \Delta u_k \text{ INSTEAD}}$

$\Delta u_k = u_k - u_{k-1}$

$u_k = \underbrace{u_{k-1}}_{\text{new extra state var.}} + \underbrace{\Delta u_k}_{\text{new „control input"}}$, x_k^u

Augment the system:

$$\begin{bmatrix} x_{k+1} \\ x_{k+1}^u \end{bmatrix} = \underbrace{\begin{bmatrix} A & B \\ 0 & I \end{bmatrix}}_{\tilde{A}} \underbrace{\begin{bmatrix} x_k \\ x_k^u \end{bmatrix}}_{\tilde{x}_k} + \underbrace{\begin{bmatrix} B \\ I \end{bmatrix}}_{\tilde{B}} \cdot \Delta u_k$$

$$y_k = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{\tilde{C}} \cdot \begin{bmatrix} x_k \\ x_k^u \end{bmatrix}$$

$$J = \frac{1}{2} \left(r_{t+N} - \tilde{C} \tilde{x}_{t+N} \right)^T S \left(r_{t+N} - \tilde{C} \tilde{x}_{t+N} \right) + \frac{1}{2} \sum_{k=0}^{N-1} \left[\left(r_{t+k} - \tilde{C} x_{t+k} \right)^T Q \left(r_{t+k} - \tilde{C} x_{t+k} \right) + \Delta u_{t+k}^T R \Delta u_{t+k} \right]$$

CONSTANT OFFSET

$$J = \cancel{\frac{1}{2} r_{t+N}^T S r_{t+N}} - r_{t+N}^T S \tilde{C} \tilde{x}_{t+N} + \frac{1}{2} \tilde{x}_{t+N}^T \tilde{C}^T S \tilde{C} \tilde{x}_{t+N}$$

$$+ \sum_{k=0}^{N-1} \left[\cancel{\frac{1}{2} r_{t+k}^T Q r_{t+k}} - r_{t+k}^T Q \tilde{C} \tilde{x}_{t+k} + \frac{1}{2} \tilde{x}_{t+k}^T \tilde{C}^T Q \tilde{C} \tilde{x}_{t+k} + \frac{1}{2} \Delta u_{t+k}^T R \Delta u_{t+k} \right]$$

$$r = \begin{bmatrix} r_t \\ r_{t+1} \\ \vdots \\ r_N \end{bmatrix} \qquad \tilde{x} = \begin{bmatrix} \tilde{x}_{t+1} \\ \vdots \\ \tilde{x}_{t+N} \end{bmatrix} \qquad \Delta u = \begin{bmatrix} \Delta u_t \\ \vdots \\ \Delta u_{t+N-1} \end{bmatrix} \qquad x_t$$

$$\min \ \frac{1}{2}\tilde{x}^T \underbrace{\begin{bmatrix} \tilde{C}^T Q \tilde{C} & & & \\ & \tilde{C}^T Q \tilde{C} & & \\ & & \ddots & \\ & & & \tilde{C}^T Q \tilde{C} \\ & & & & \tilde{C}^T S \tilde{C} \end{bmatrix}}_{=\bar{\bar{Q}}} \tilde{x} - r^T \underbrace{\begin{bmatrix} Q\tilde{C} & & & \\ & Q\tilde{C} & & \\ & & \ddots & \\ & & & S\tilde{C} \end{bmatrix}}_{=\bar{\bar{T}}} \tilde{x} + \frac{1}{2}\Delta u^T \underbrace{\begin{bmatrix} R & & \\ & \ddots & \\ & & R \end{bmatrix}}_{=\bar{\bar{R}}} \Delta u$$

! $\tilde{C}$ was missing in the video

$$\underbrace{\text{s.t.} \quad \tilde{X} = \underbrace{\begin{bmatrix} 0 & & & \\ \tilde{A} & 0 & & \\ & \tilde{A} & \ddots & \\ & & \ddots & \ddots \\ & & \tilde{A} & 0 \end{bmatrix}}_{=\bar{\bar{A}}} \tilde{X} + \underbrace{\begin{bmatrix} \tilde{B} & & \\ & \ddots & \\ & & \tilde{B} \end{bmatrix}}_{=\bar{\bar{B}}} \Delta u + \underbrace{\begin{bmatrix} \tilde{A} \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{=\bar{\bar{A}}_0} \tilde{x}_t}_{\text{SIMULTANEOUS } (\tilde{X} \ \& \ \Delta u)}$$

ELIMINATE $\tilde{X}$ (EXPRESS AS FUNCTION OF Δu and $\tilde{x}_t$)

$$\tilde{X} = \underbrace{\begin{bmatrix} \tilde{B} & & & \\ \tilde{A}\tilde{B} & \tilde{B} & & \\ \tilde{A}^2\tilde{B} & \tilde{A}\tilde{B} & \tilde{B} & \\ \vdots & & & \ddots \\ \tilde{A}^{N-1}\tilde{B} & \cdots & & \tilde{B} \end{bmatrix}}_{=\bar{\bar{C}}} \cdot \Delta u + \underbrace{\begin{bmatrix} \tilde{A} \\ \tilde{A}^2 \\ \vdots \\ \tilde{A}^N \end{bmatrix}}_{=\hat{\hat{A}}} \tilde{x}_t$$

COST

$$J = \frac{1}{2}\left(\bar{\bar{C}}\Delta u + \hat{\hat{A}}\tilde{x}_t\right)^T \bar{\bar{Q}} \left(\bar{\bar{C}}\Delta u + \hat{\hat{A}}\tilde{x}_t\right) + \frac{1}{2}\Delta u^T \bar{\bar{R}} \Delta u - r^T \bar{\bar{T}} \left(\bar{\bar{C}}\Delta u + \hat{\hat{A}}\tilde{x}_t\right)$$

$$= \ldots$$

$= \ldots$ ignoring constant terms

$$= \frac{1}{2}\Delta u^T \underbrace{\left(\bar{\bar{C}}^T \bar{\bar{Q}} \bar{\bar{C}} + \bar{\bar{R}}\right)}_{=\bar{\bar{H}}} \Delta u + [\tilde{x}_t^T \ \ r^T] \cdot \underbrace{\begin{bmatrix} \hat{\hat{A}}^T \bar{\bar{Q}} \bar{\bar{C}} \\ -\bar{\bar{T}} \bar{\bar{C}} \end{bmatrix}}_{=\bar{\bar{F}}^T} \cdot \Delta u$$

$$\boxed{= \frac{1}{2}\Delta u^T \cdot \bar{\bar{H}} \cdot \Delta u + [\tilde{x}_t^T \ \ r^T] \cdot F^T \cdot \Delta u}$$

$$\Downarrow \nabla_{\Delta u}$$

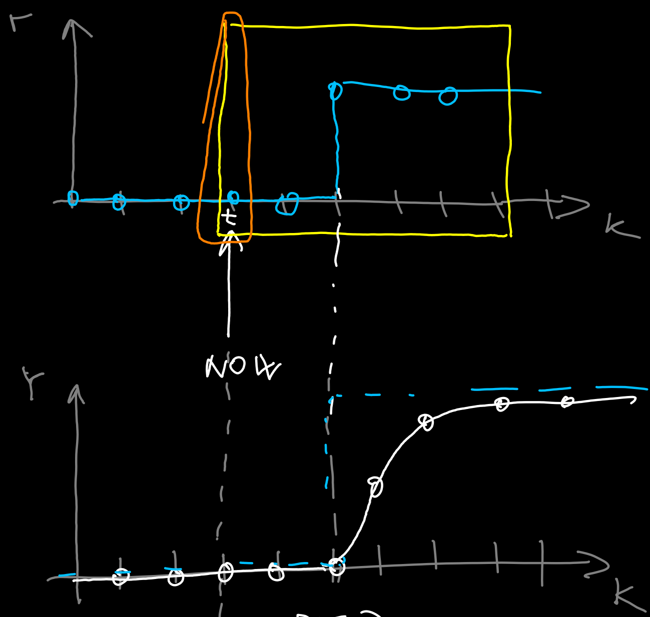
$$\boxed{\nabla J = \bar{\bar{H}} \Delta u + \bar{\bar{F}} \cdot \begin{bmatrix} \tilde{x}_t \\ r \end{bmatrix} = 0}$$

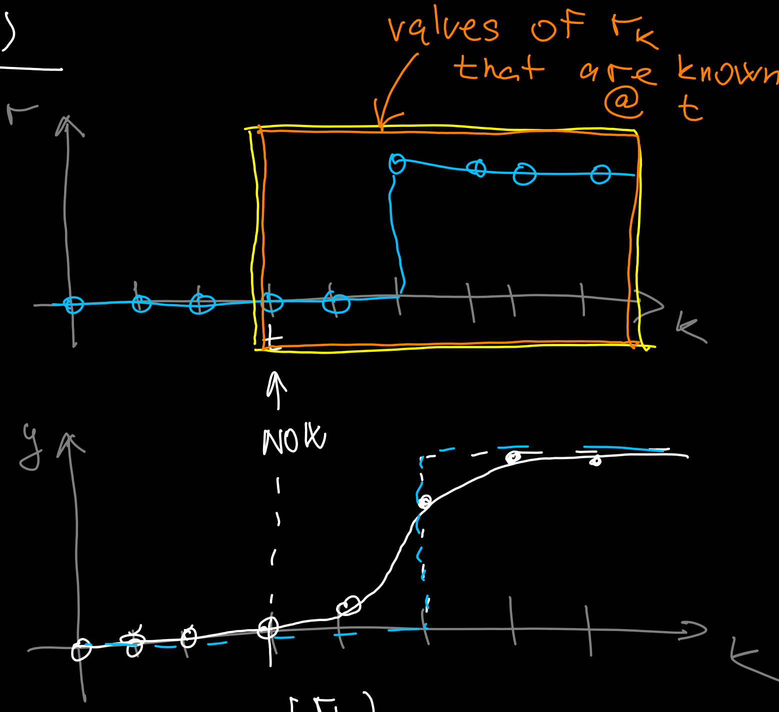
adding constraints:

$\leq u_k \leq$

$\leq x_k \leq$ } use $\bar{\bar{C}}$ and $\hat{\hat{A}}$

ANTICIPATION (PREVIEW)





$$r = \begin{bmatrix} r_t \\ r_t \\ \vdots \\ r_t \end{bmatrix} \quad \text{vs.} \quad r = \begin{bmatrix} r_t \\ r_{t+1} \\ \vdots \\ r_{t+N} \end{bmatrix}$$