1 a) $\frac{\partial f}{\partial x} = e^{2y} + y\cos x \qquad (1)$

$\frac{\partial f}{\partial y} = \sin x + 2xe^{2y} - y \qquad (2)$

$(1) \Rightarrow f(x,y) = xe^{2y} + y\sin x + C(y)$

Dosazením do (2) dostaneme

$$\cancel{2xe^{2y}} + \cancel{\sin x} + C'(y) = \cancel{\sin x} + \cancel{2xe^{2y}} - y$$

$$C'(y) = -y \Rightarrow C(y) = -\frac{y^2}{2} + K, \text{ kde } K \in \mathbb{R}$$

Z $f(0,1) = 0$ máme $0 = C(1) = -\frac{1}{2} + K \Rightarrow K = \frac{1}{2}$.

Hledaný potenciál proto je:

$$f(x,y) = xe^{2y} + y\sin x - \frac{y^2}{2} + \frac{1}{2}$$

b) Protože F je potenciálové, je

$$\int_c F(x,y)\cdot ds = f(\varphi(1)) - f(\varphi(0)) = f(0,0) - f(0,1)$$

$$\overset{a)}{=} \frac{1}{2}$$

② i) Podezřelé body v int M:

$$0 = \frac{\partial f}{\partial x} = 8x - 2 \quad \dots \quad x = \frac{1}{4}$$

$$0 = \frac{\partial f}{\partial y} = 6y \quad \dots \quad y = 0$$

(Bod $(\frac{1}{4}, 0)$ leží v int M.)

ii) Podezřelé body v ∂M:

$$\begin{aligned} 8x - 2 + 2\lambda x &= 0 \\ 6y + 2\lambda y &= 0 \quad \dots \quad 2y(3+\lambda) = 0 \begin{cases} y = 0 \\ \lambda = -3 \end{cases} \\ x^2 + y^2 &= 1 \end{aligned}$$

- $y = 0 \xRightarrow{x^2+y^2=1} x = \pm 1$
- $\lambda = -3 \xRightarrow{8x-2+2\lambda x=0} 2x - 2 = 0 \quad \dots \quad x = 1$

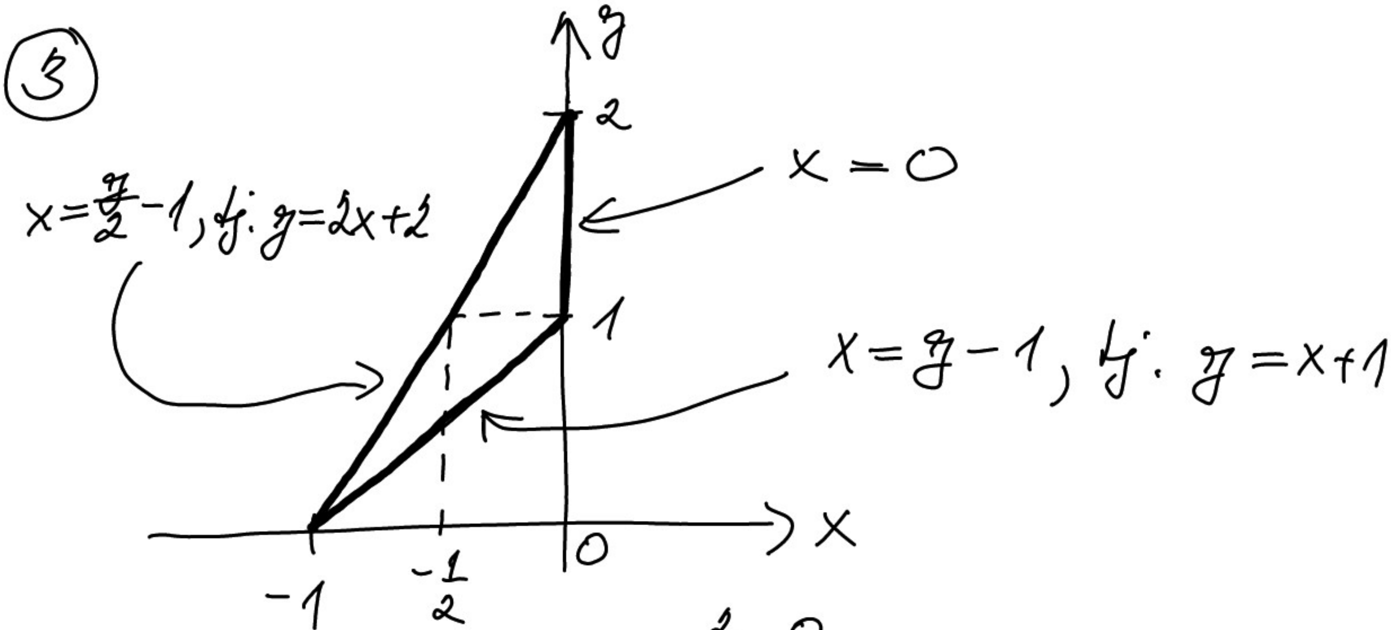
Odtud $y = 0$.

Podezřelé body z extrému jsou $(\frac{1}{4}, 0)$, $(1, 0)$ a $(-1, 0)$. Díky Weierstrassově větě víme, že body min. a max. existují. Protože

$$\left. \begin{aligned} f(\tfrac{1}{4}, 0) &= \tfrac{1}{4} - \tfrac{1}{2} = -\tfrac{1}{4} \\ f(1, 0) &= 2 \\ f(-1, 0) &= 6 \end{aligned} \right\} \Rightarrow$$

$(\frac{1}{4}, 0)$ je jediný bod minima f na M a $(-1, 0)$ je jediný bod maxima f na M.

3



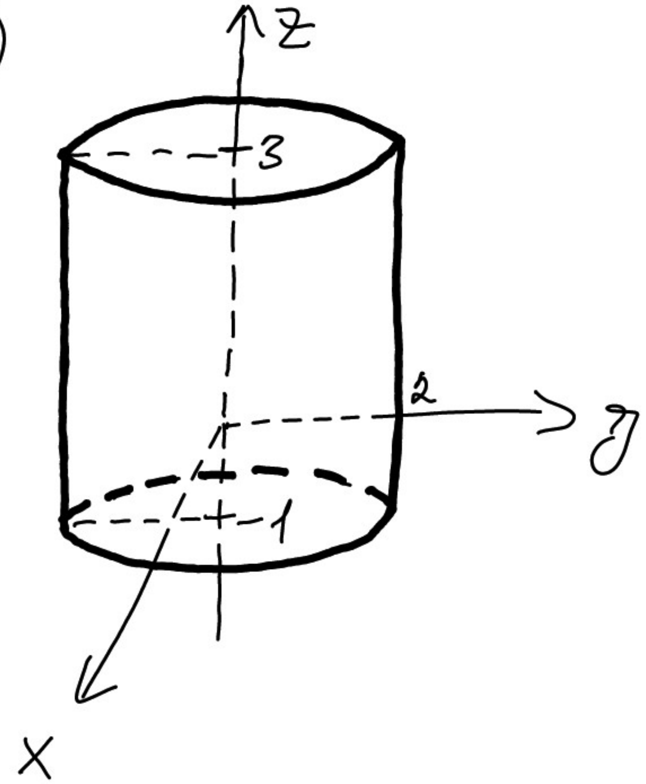
$$\int_0^1 \int_{\frac{y}{2}-1}^{y-1} e^{\frac{x^2}{2}+x}\,dx\,dy + \int_1^2 \int_{\frac{y}{2}-1}^{0} e^{\frac{x^2}{2}+x}\,dx\,dy =$$

$$= \int_{-1}^{0} \int_{x+1}^{2x+2} e^{\frac{x^2}{2}+x}\,dy\,dx = \int_{-1}^{0} \left[2x+2-(x+1)\right] e^{\frac{x^2}{2}+x}\,dx$$

$$= \int_{-1}^{0} (x+1)\, e^{\frac{x^2}{2}+x}\,dx = \left[e^{\frac{x^2}{2}+x}\right]_{-1}^{0}$$

$$= 1 - e^{-\frac{1}{2}}$$

4



$$\int_S F(x,y,z)\cdot d\sigma = \int_\Omega \overbrace{\nabla\cdot F(x,y,z)}^{2(z-1)}\, d\lambda_3$$

$$= \int_{-1}^{3}\int_{0}^{2\pi}\int_{0}^{2} 2(z-1)\, r\, dr\, d\varphi\, dz$$

$$= 4\pi \int_{-1}^{3} \left[\frac{r^2}{2}\right]_0^2 (z-1)\, dz$$

$$= 8\pi \left[\frac{(z-1)^2}{2}\right]_{-1}^{3} = 8\pi(2-2) = \underline{\underline{0}}$$

⑤ a)

$$f(x)=\frac{1}{2(x+1)+3}=\frac{1}{3}\,\frac{1}{1+\frac{2}{3}(x+1)}=\frac{1}{3}\,\frac{1}{1-\left[-\frac{2}{3}(x+1)\right]}$$

$$=\frac{1}{3}\sum_{n=0}^{\infty}\left(-\frac{2}{3}\right)^n(x+1)^n=\sum_{n=0}^{\infty}\frac{(-2)^n}{3^{n+1}}(x+1)^n$$

pro $\left|\frac{2}{3}(x+1)\right|<1$, tj. pro $|x+1|<\frac{3}{2}$.

(Polom. konv. je tedy $R=\frac{3}{2}$.)

b) $$f'(x)=\left[\sum_{n=0}^{\infty}\frac{(-2)^n}{3^{n+1}}(x+1)^n\right]'$$

$$=\sum_{n=1}^{\infty}\frac{(-2)^n n}{3^{n+1}}(x+1)^{n-1}$$

pro $|x+1|<\frac{3}{2}$.

(Polom. konv. je opět $R=\frac{3}{2}$.)