ŘEŠENÍ

① $\frac{\partial f}{\partial x} = \sin(y^2 - x) - x\cos(y^2 - x) + \frac{1}{y}$

$\frac{\partial f}{\partial y} = 2xy\cos(y^2 - x) - \frac{x}{y^2}$

$\Rightarrow \frac{\partial f}{\partial x}(-1,1) = \sin 2 + \cos 2 + 1 =: \alpha$

$\frac{\partial f}{\partial y}(-1,1) = -2\cos 2 + 1 =: \beta$

Tedy

$0 = D_h f(-1,1) = \alpha h_1 + \beta h_2 \Rightarrow h_2 = -\frac{\alpha}{\beta} h_1$

$1 = \|h\|^2 = h_1^2 + h_2^2 = h_1^2 + \frac{\alpha^2}{\beta^2} h_1^2 = \left(1 + \frac{\alpha^2}{\beta^2}\right) h_1^2$

$= \frac{\alpha^2 + \beta^2}{\beta^2} h_1^2 \Rightarrow h_1 = \pm \frac{\beta}{\sqrt{\alpha^2 + \beta^2}}$

$\Rightarrow h = \left(\frac{\beta}{\sqrt{\alpha^2+\beta^2}}, -\frac{\alpha}{\sqrt{\alpha^2+\beta^2}}\right)$

nebo $h = \left(-\frac{\beta}{\sqrt{\alpha^2+\beta^2}}, \frac{\alpha}{\sqrt{\alpha^2+\beta^2}}\right)$.

2) i) $(x,y) \in \operatorname{int} M$

$$\frac{\partial f}{\partial x} = 2x-3=0 \qquad \iff \quad x=\frac{3}{2},\ y=0$$

$$\frac{\partial f}{\partial y} = -2y=0 \qquad \text{Zřejmě } \left(\tfrac{3}{2},0\right)\in \operatorname{int} M$$

ii) $(x,y)\in \partial M$

$$2x-3+\lambda 2x=0 \quad (1)$$

$$-2y+\lambda 4y=0 \quad (2) \quad \ldots 2y(-1+2\lambda)=0$$

$$x^2+2y^2=9 \quad (3)$$

- $y=0 \overset{(3)}{\Longrightarrow} x=\pm 3$
- $\lambda=\frac{1}{2} \overset{(1)}{\Longrightarrow} 3x-3=0 \Rightarrow x=1$

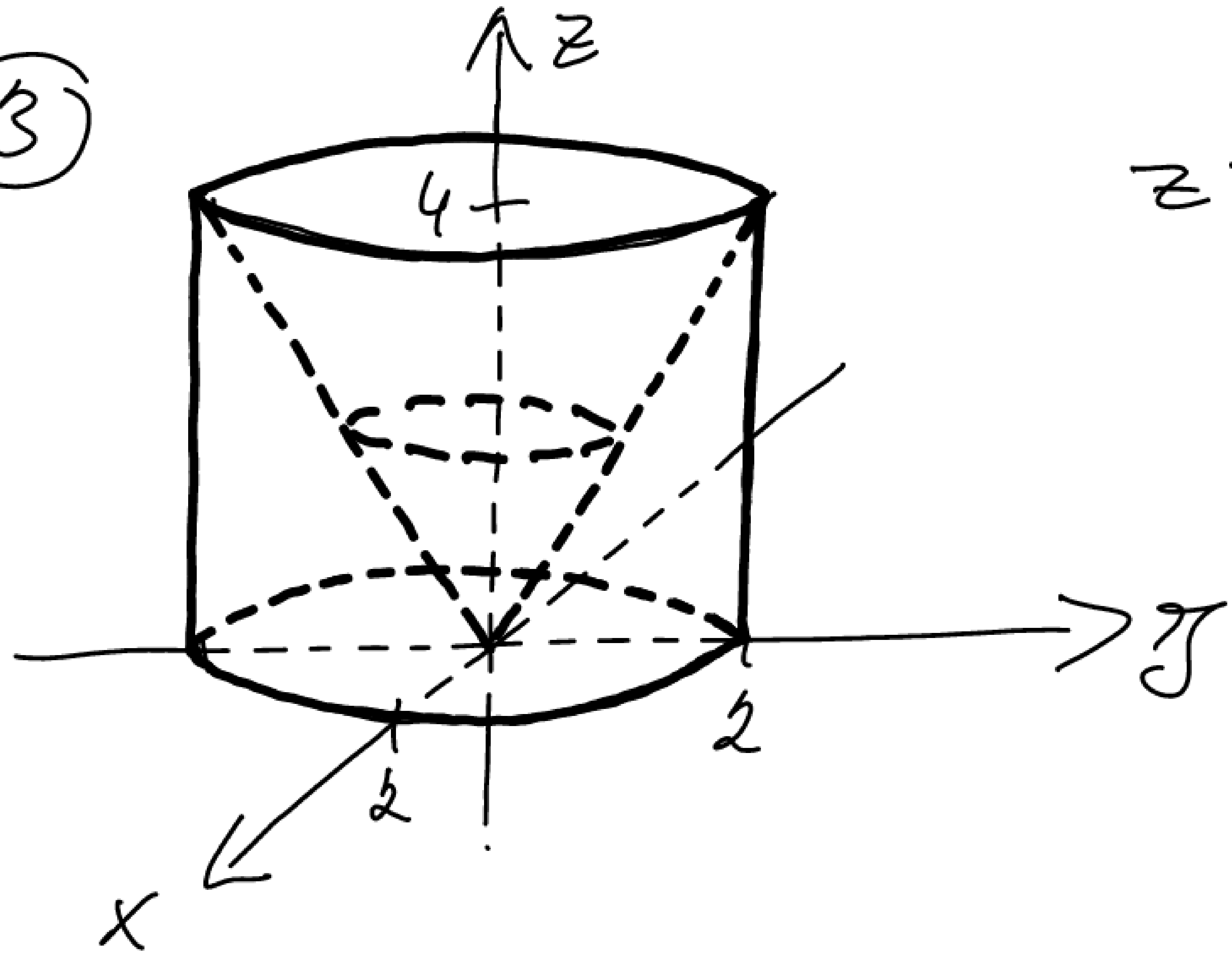
Z (3) máme $y^2=4 \Rightarrow y=\pm 2$

Podezřelé body z extrému: $A=\left(\frac{3}{2},0\right)$, $B=(3,0)$, $C=(-3,0)$, $D=(1,2)$, $E=(1,-2)$

$$\left.\begin{aligned} f(A)&=\frac{9}{4}-\frac{9}{2}=-\frac{9}{4}\\ f(B)&=0\\ f(C)&=18\\ f(D)&=1-4-3=-6\\ f(E)&=-6\end{aligned}\right\} \xRightarrow{\text{Weierstrassova věta}} C \text{ je bod maxima,}$$

D, E jsou body minima.

③



$z \geq 0, \quad z^2 \leq 4(x^2+y^2) \leq 16$

$\Rightarrow \quad 0 \leq z \leq 2r \leq 4$

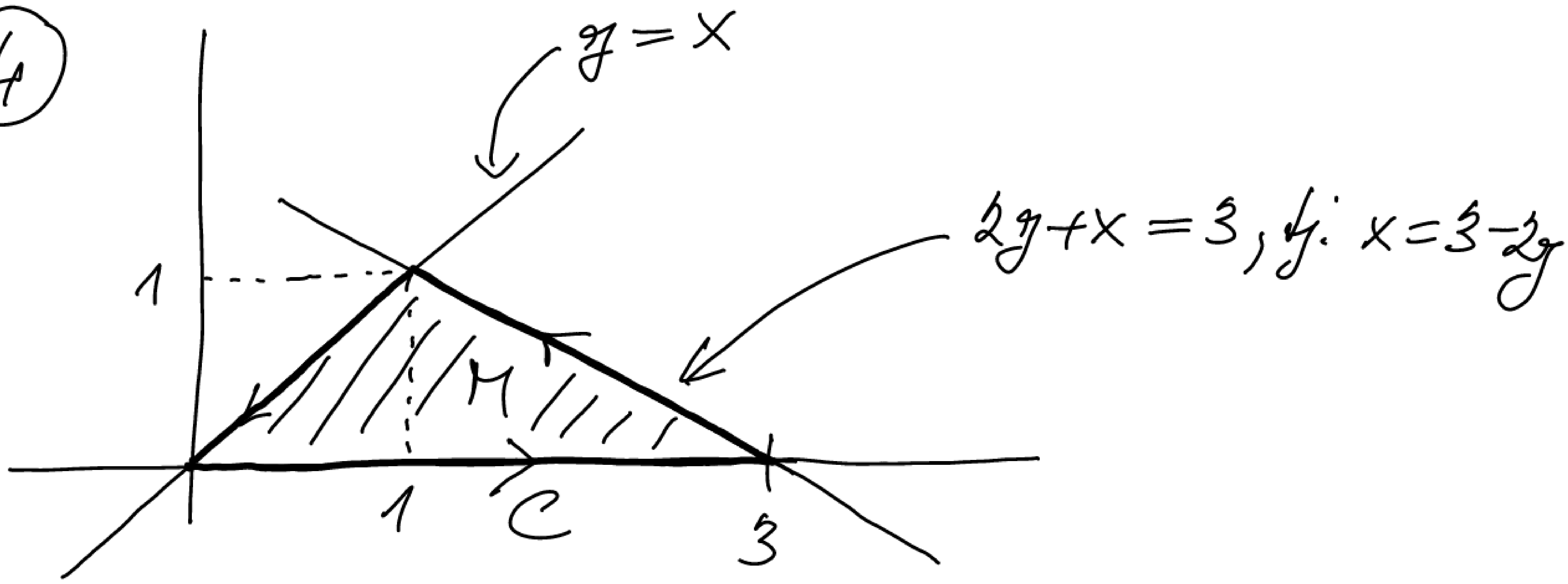
$$\int_M x^2+y^2 \, d\lambda_3 = \int_0^{2\pi}\int_0^{2}\int_0^{2r} r^2 r \, dz\, dr\, d\varphi$$

$$= \int_0^{2\pi}\int_0^{2} 2r^4 \, dr\, d\varphi$$

$$= \int_0^{2\pi} \frac{2}{5} 2^5 \, d\varphi = 2\pi \frac{64}{5}$$

$$= \frac{128}{5}\pi .$$

4



$$\int_C F \cdot ds = \int_M \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \, d\lambda_2$$

$$= \int_0^1 \int_y^{3-2y} y - 2y \, dx \, dy$$

$$= \int_0^1 -y(3-2y-y) \, dy$$

$$= \int_0^1 -3y + 3y^2 \, dy = -\frac{3}{2} + 1$$

$$= -\frac{1}{2}.$$

(5) a)

$$\lim_{k\to\infty} \sqrt[k]{\frac{1}{4^k}|x+1|^{2k}} = \frac{1}{4}|x+1|^2 < 1 \iff |x+1| < 2$$

Poloměr konvergence je $R=2$.

Zadaná řada nekonverguje v bodě $x=2$, protože $|2+1|>2$.

b) $$\sum_{k=0}^{\infty}{}' \frac{(k+2)(-1)^k}{k!} x^{k+1} = \left(\sum_{k=0}^{\infty}{}' \frac{(-1)^k}{k!} x^{k+2}\right)'$$

$$= \left(x^2 \sum_{k=0}^{\infty}{}' \frac{(-x)^k}{k!}\right)' = \left(x^2 e^{-x}\right)'$$

$$= (2x - x^2)e^{-x} \quad \text{pro } x \in \mathbb{R}.$$