ŘEŠENÍ

(1) $$\frac{\partial f}{\partial x} = 2x + \frac{y}{x+2y}$$

$$\frac{\partial f}{\partial y} = \ln(x+2y) + \frac{2y}{x+2y}$$

$$\Rightarrow \quad \frac{\partial f}{\partial x}(-1,1) = -1, \qquad \frac{\partial f}{\partial y}(-1,1) = 2$$

a) $$v = \frac{1}{\|\nabla f(-1,1)\|}\nabla f(-1,1) = \frac{1}{\sqrt{5}}(-1,2)$$

b) $$0 = \nabla_h f(-1,1) = h \cdot \nabla f(-1,1) \overset{h=(h_1,h_2)}{=} -h_1 + 2h_2$$

$$\Rightarrow h_1 = 2h_2 .$$

Protože $1 = \|h\|^2 = h_1^2 + h_2^2 \overset{h_1=2h_2}{=} 5h_2^2$,

je $h_2 = \pm\frac{1}{\sqrt{5}}$. Tedy

$$h = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right) \text{ nebo } h = \left(-\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right).$$

(2) $\frac{\partial f}{\partial x} = 2x - 2y - 3 = 0 \implies y^2 - 2y - 3 = 0$

$\frac{\partial f}{\partial y} = y^2 - 2x = 0 \implies y^2 = 2x$

$\frac{\partial f}{\partial z} = 2z + 2 = 0 \implies z = -1$

Protože $y^2 - 2y - 3 = 0 \iff y = \frac{2 \pm \sqrt{4+12}}{2} = \begin{cases} 3 \\ -1 \end{cases}$,

stacionární body jsou $A = \left(\frac{9}{2}, 3, -1\right)$

$B = \left(\frac{1}{2}, -1, -1\right)$

$$H_f = \begin{pmatrix} 2 & -2 & 0 \\ -2 & 2y & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

- $H_f(A) = \begin{pmatrix} 2 & -2 & 0 \\ -2 & 6 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ je pozit. def., neboť

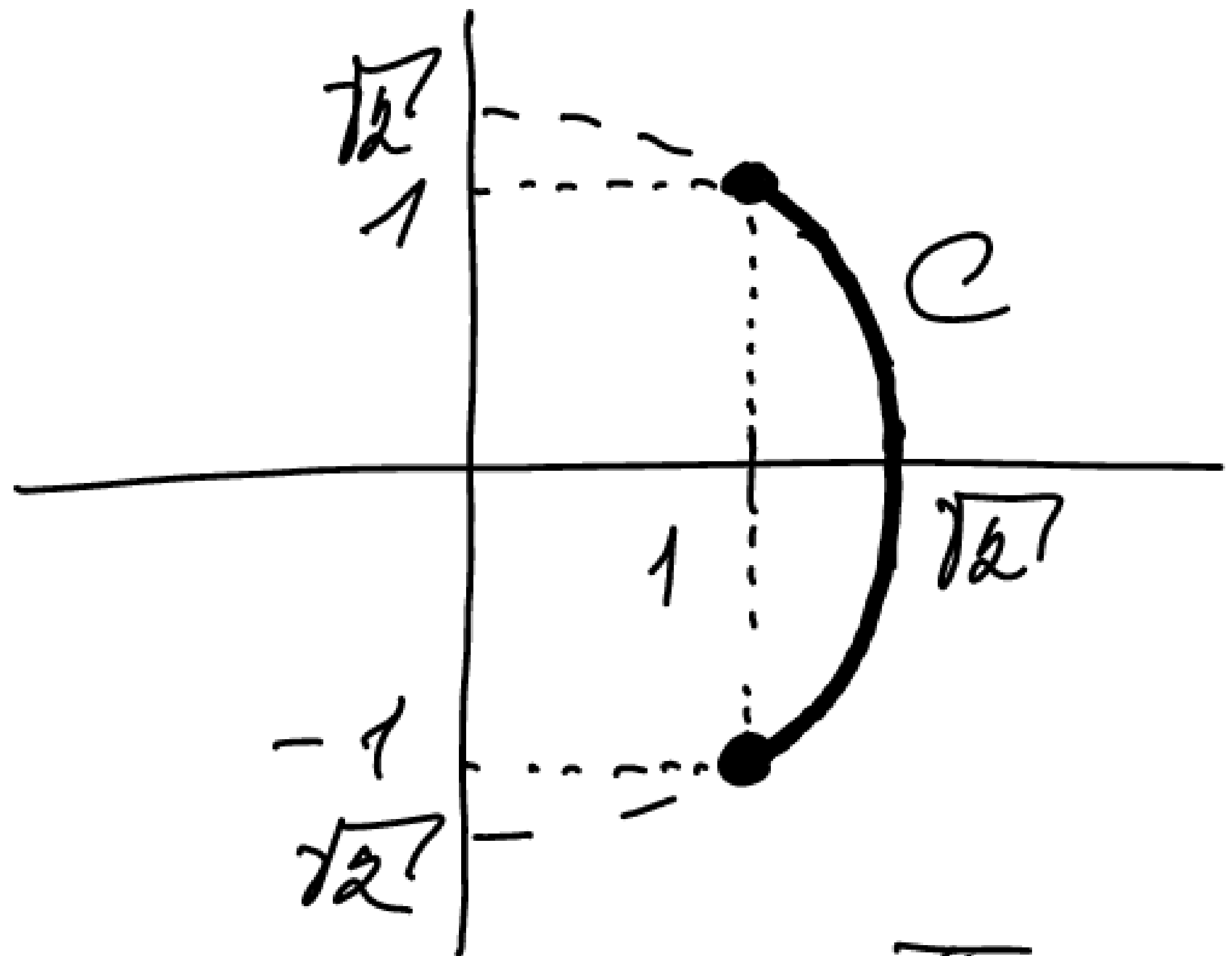
$2 > 0$, $\begin{vmatrix} 2 & -2 \\ -2 & 6 \end{vmatrix} = 8 > 0$ a $\det H_f(A) = 2 \begin{vmatrix} 2 & -2 \\ -2 & 6 \end{vmatrix} > 0$

$\implies$ A je bod lokálního minima.

- $H_f(B) = \begin{pmatrix} 2 & -2 & 0 \\ -2 & -2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ je indefinitní,

neboť $\begin{vmatrix} 2 & -2 \\ -2 & -2 \end{vmatrix} = -8 < 0 \implies$ B je sedlový bod.

③



$$C: \varphi(t) = (\sqrt{2}\cos t, \sqrt{2}\sin t),$$
$$t \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$$

$$\|\varphi'(t)\| = \sqrt{2}$$

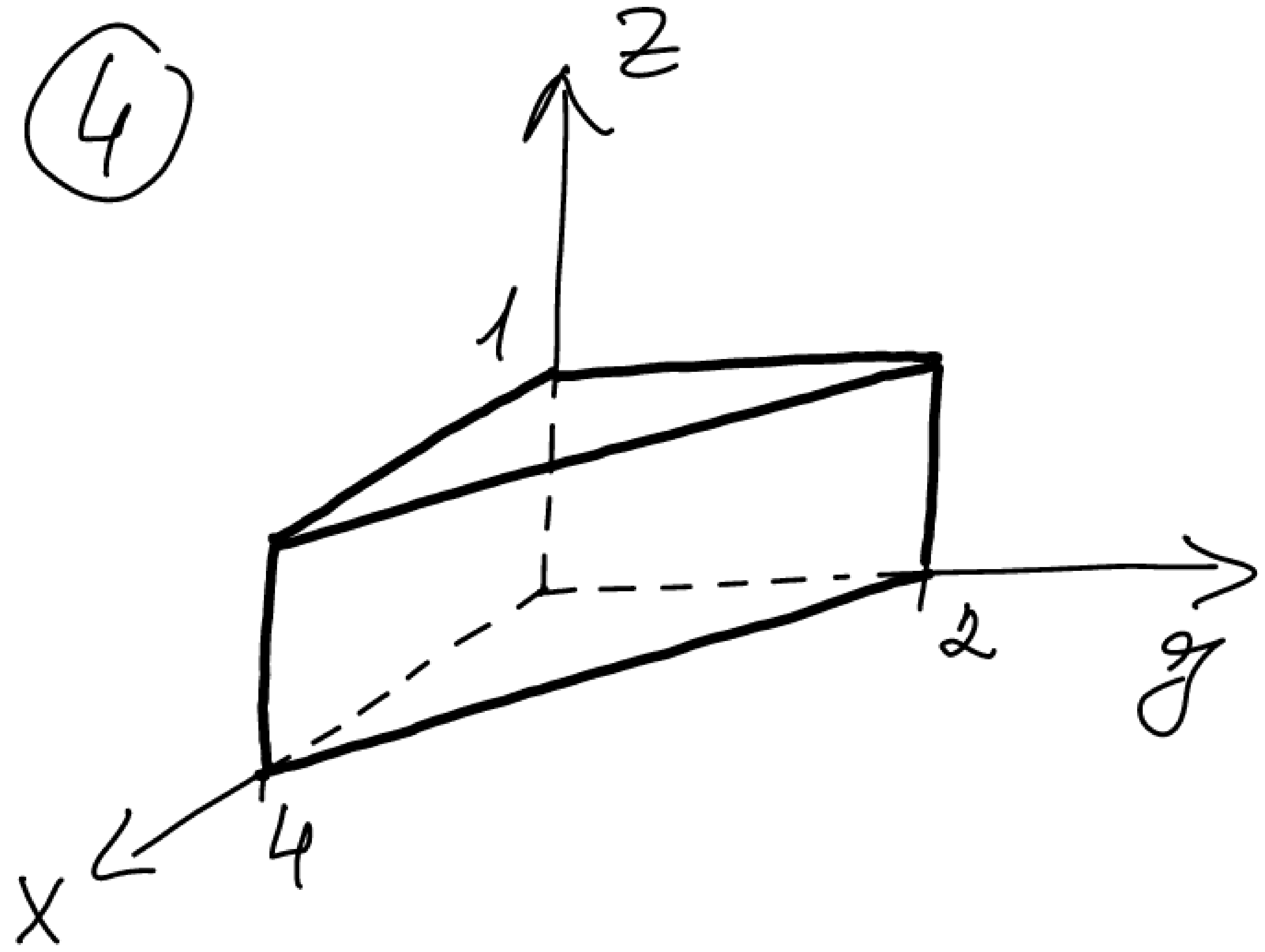
$$\int_C y - xy \, ds = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\sqrt{2}\cos t - 2\sin t \cos t\right)\sqrt{2}\, dt$$

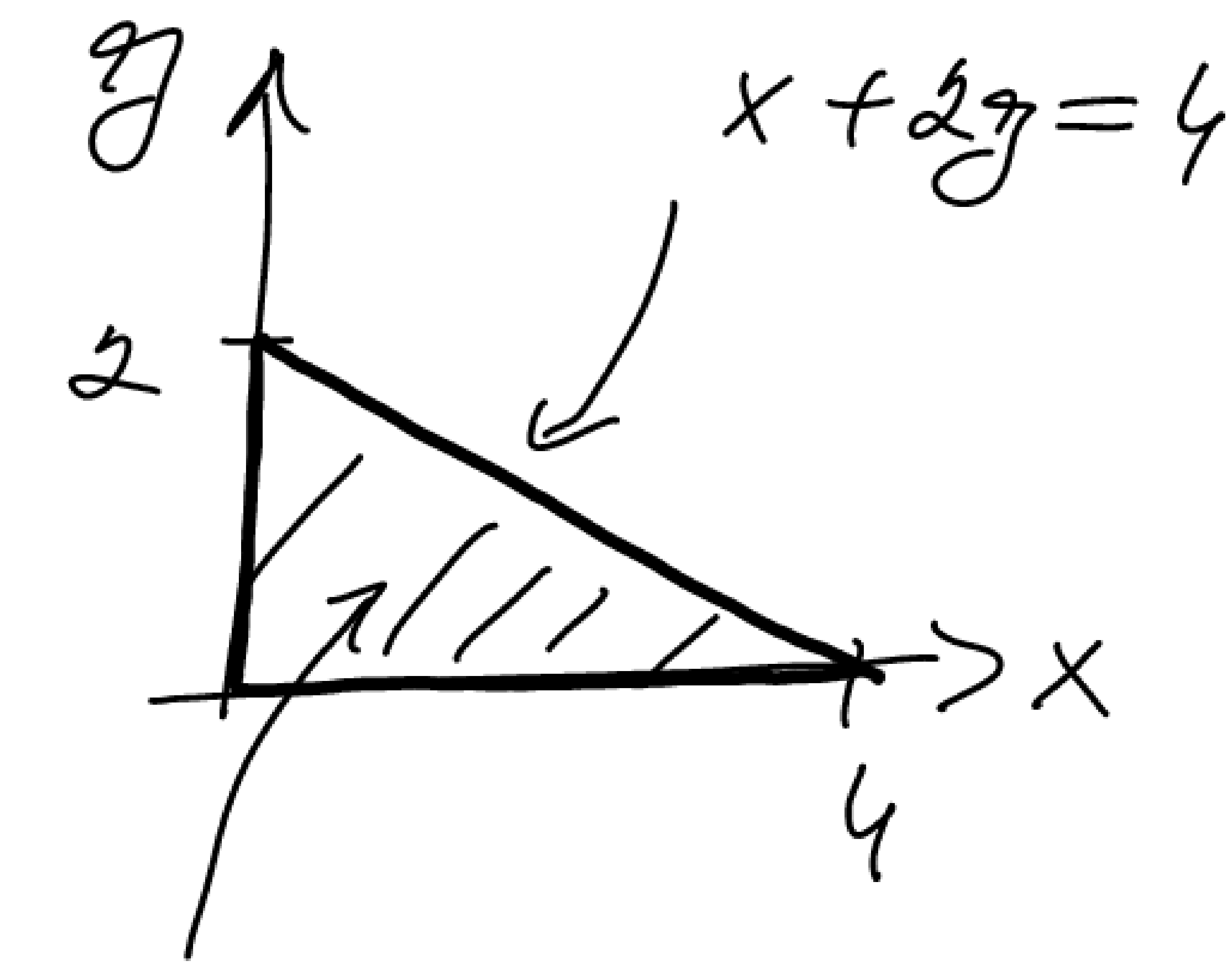
$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 2\cos t - \sqrt{2}\sin(2t)\, dt$$

$$= 4\int_0^{\frac{\pi}{4}} \cos t \, dt = 4\left[\sin t\right]_0^{\frac{\pi}{4}}$$

$$= 2\sqrt{2}.$$

④





$M^{[z]}$ pro $z \in [0,1]$

$$\nabla \cdot F = -yz + 2yz + 2yz = 3yz$$

$$\Rightarrow \int_S F \cdot d\sigma = \int_M \nabla \cdot F \, d\lambda_3 = \int_0^1 \int_{M^{[z]}} 3yz \, d\lambda_2(x,y) \, dz$$

$$= \int_0^1 \int_0^2 \int_0^{4-2y} 3yz \, dx \, dy \, dz$$

$$= 6 \int_0^1 \int_0^2 yz(2-y) \, dy \, dz$$

$$= 6 \left[y^2 - \frac{y^3}{3} \right]_0^2 \int_0^1 z \, dz$$

$$= 6 \left(4 - \frac{8}{3} \right) \cdot \frac{1}{2} = 12 - 8 = 4.$$

5 a)

$$\frac{1}{2-x} = \frac{1}{-1-(x-3)} = -\frac{1}{1+(x-3)} = -\frac{1}{1-[-(x-3)]}$$

$$= -\sum_{k=0}^{\infty}(-1)^k(x-3)^k = \sum_{k=0}^{\infty}(-1)^{k+1}(x-3)^k$$

pro $|x-3|<1$ (tj: $R=1$).

b)

$$f'(x) = \left(\frac{1}{2-x}\right)' \overset{a)}{=} \left(\sum_{k=0}^{\infty}(-1)^{k+1}(x-3)^k\right)'$$

$$= \sum_{k=1}^{\infty}(-1)^{k+1}k(x-3)^{k-1}$$

pro $|x-3|<1$ (tj: $R=1$).