# Automatic control (AC) course. Basic info.

- location & time

    lectures Mo and We, starting 9:15, room K24

    labs We, starting 11:00, room K26

- web support

    https://moodle.dce.fel.cvut.cz/course/view.php?id=57

- AC team

    Martin Hromcik m.hromcik@c-a-k.cz, +420 608 665 391
    Ivo Herman, Dan Martinec

- exam

    written + oral + semestral projects, homework, …

- literature

    Franklin, Powell, Emami-Naeini: Feedback Control of Dynamics Systems.

- SW

    Matlab and related …

# Control systems: nomenclature

- Systems and models (example: ACFA 2020, www.acfa2020.eu)

system / object
(BWB A/C ACFA2020)

model II: set of ODEs.
(rigid body flight dynamics)

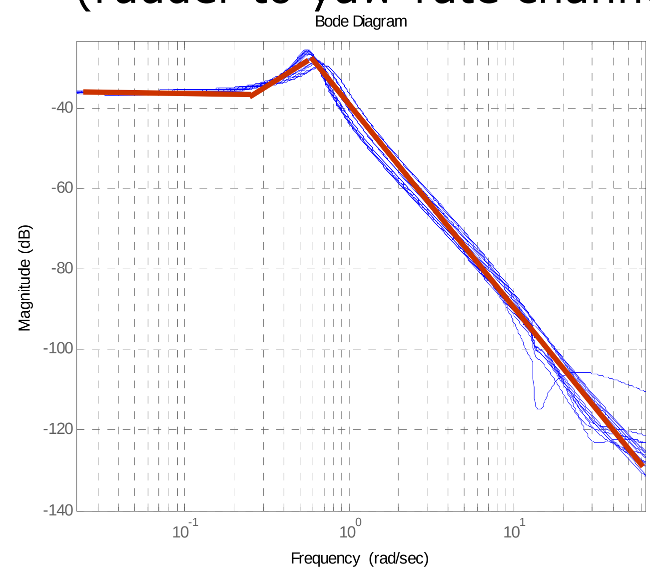
$$\dot{v} + a_{11}v + a_{12}\alpha + a_{13}\theta = c_{11}\delta_T$$

$$a_{21}v + \dot{\alpha} + a_{22}\alpha - \dot{\theta} + a_{23}\theta = c_{22}\delta_V$$

$$a_{31}v + a_{30}\dot{\alpha} + a_{32}\alpha + \ddot{\theta} + a_{33}\dot{\theta} = c_{32}\delta_V$$

model I: FEM.
(illustration of the mesh grid)

model III: frequency responses.
(rudder to yaw-rate channel)

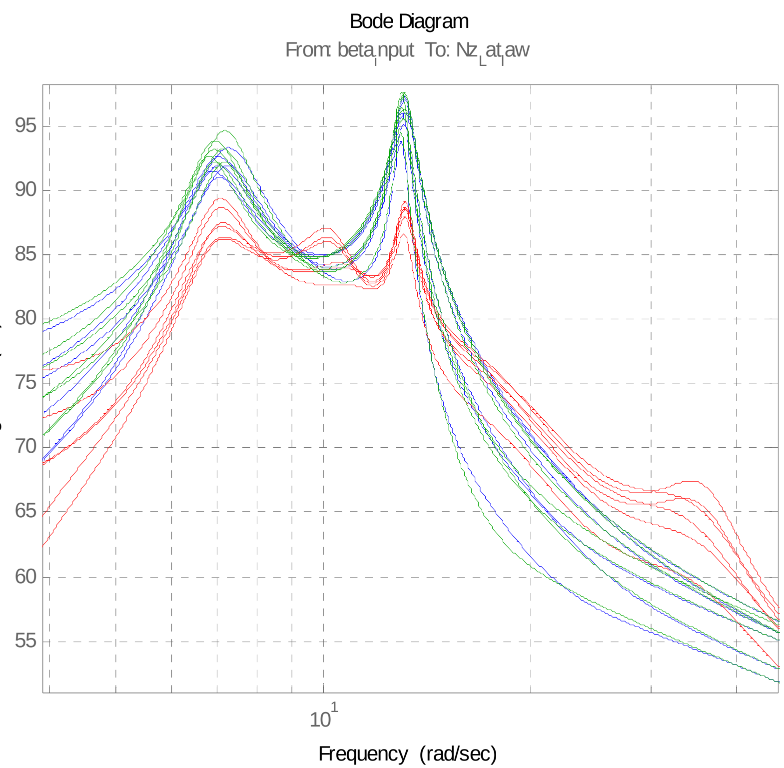



- high-complexity: used for control systems performance assessment

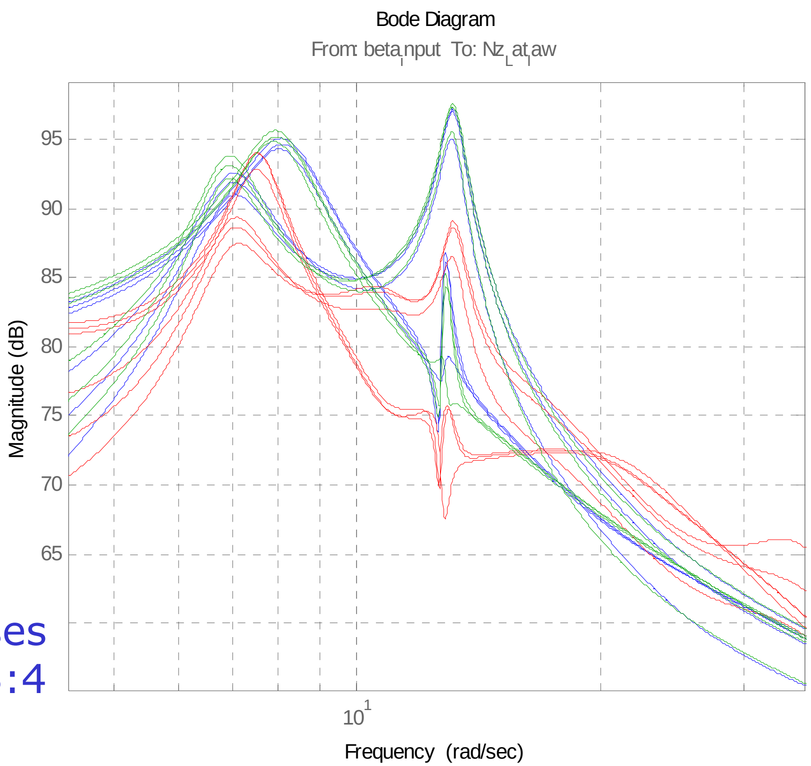
- low-complexity: used for control systems design

# Control systems: contributions and goals

- dynamics modification (ACFA 2020 continued ...)




mass cases
1:3,4:6

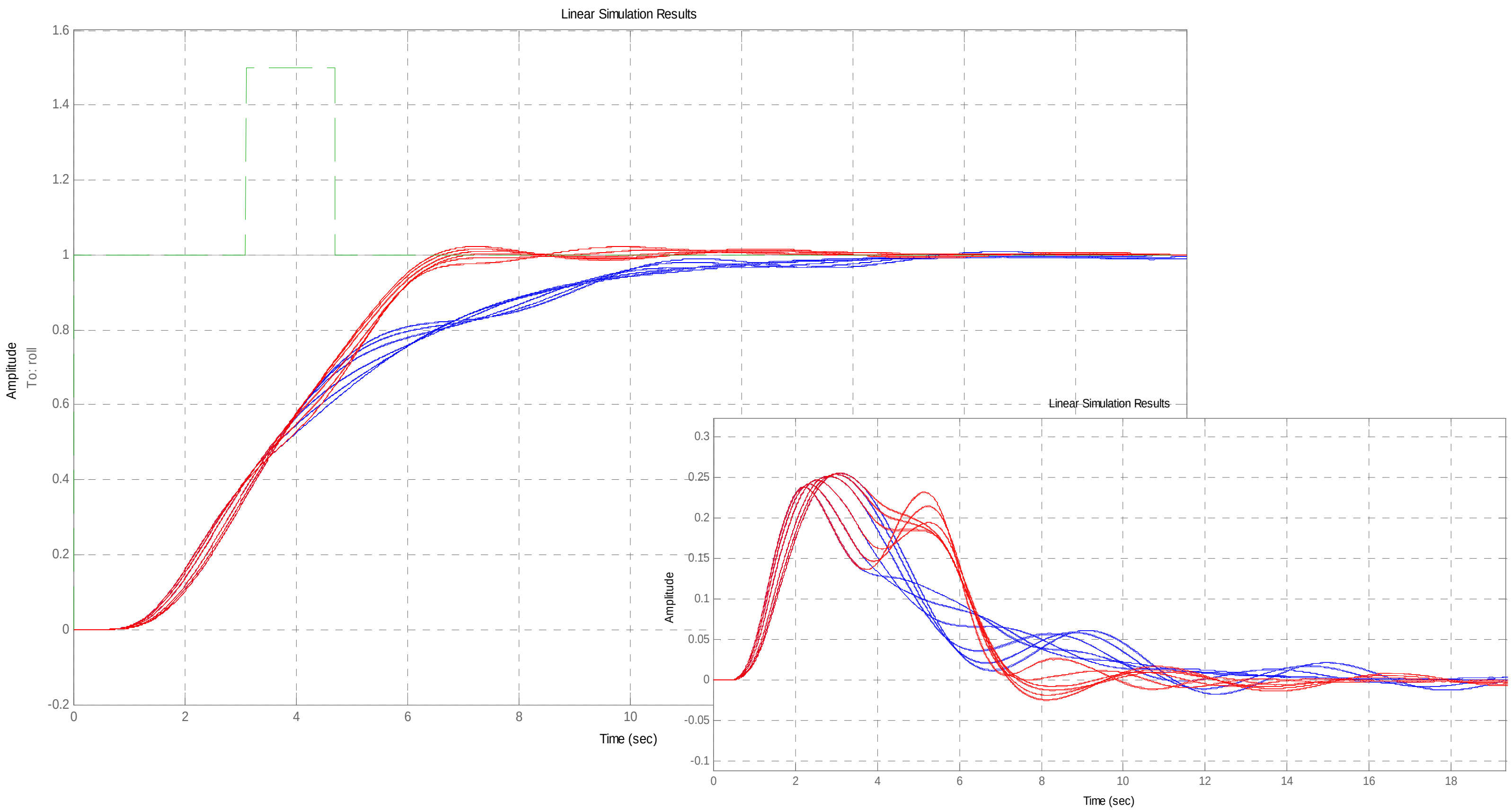



mass cases
1:3,3:4

- the rudder channel (open-loop, feedback)
- robust for 12 PAX/FUEL cases
- reduction by >5dB for all cases

# Control systems: contributions and goals

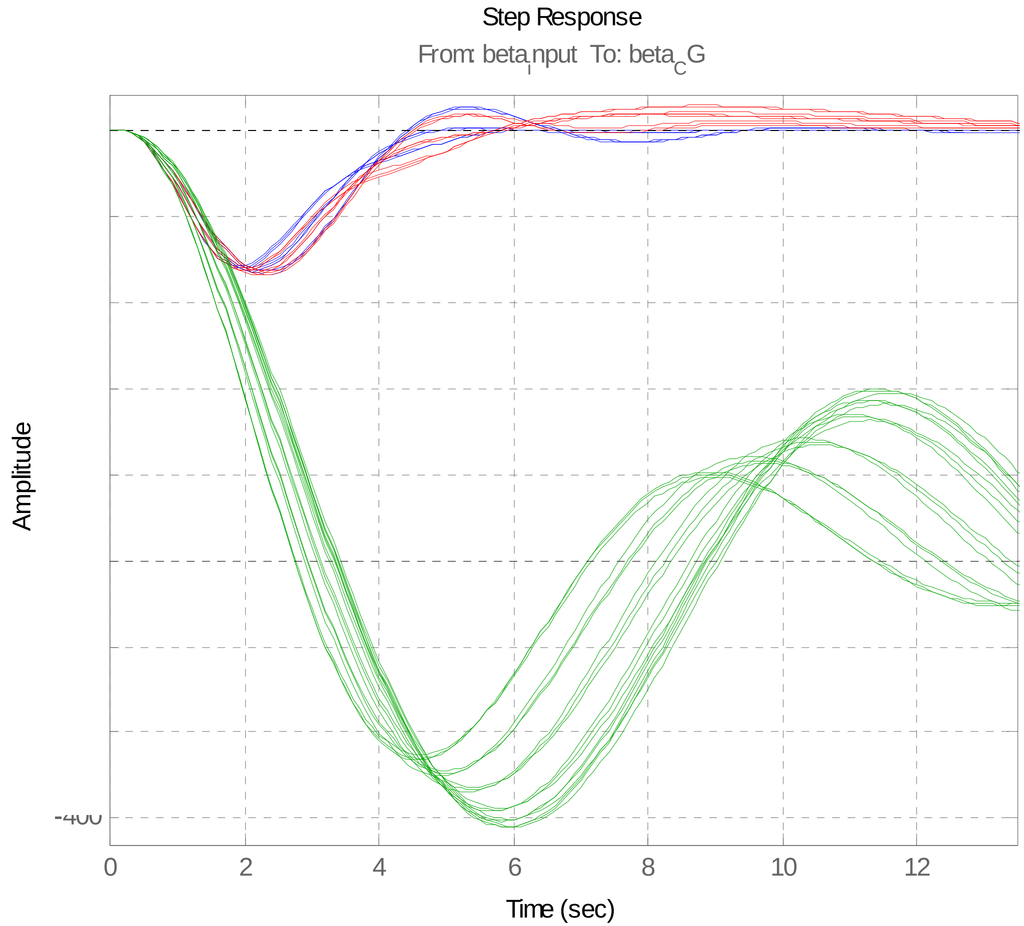
- reference command tracking (ACFA 2020 continued ...)




- response from roll angle setpoint to roll angle (ROLL AP only, complete FB / FF)
- roll-rate

# Control systems: contributions and goals

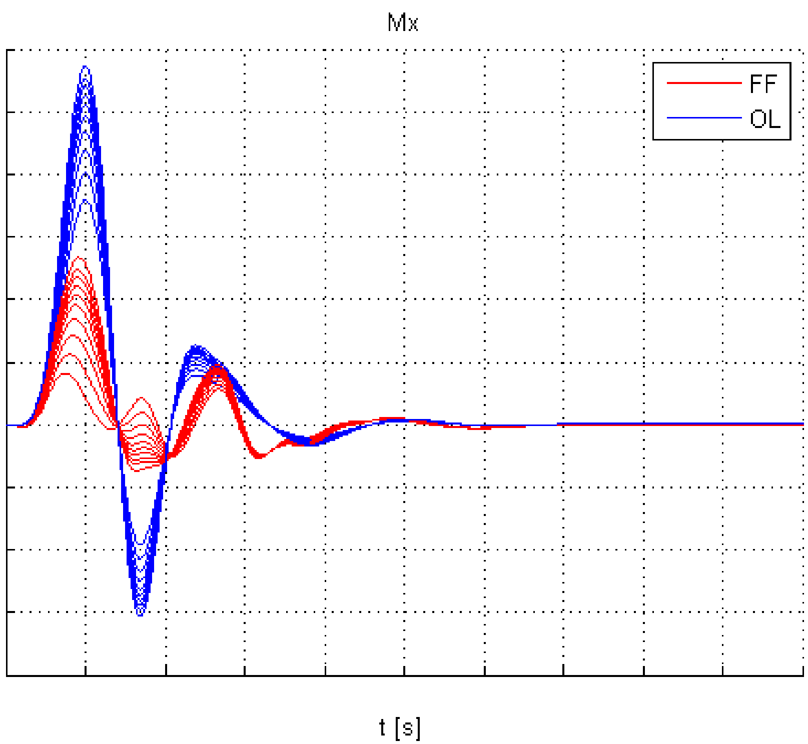
- unmeasurable disturbance attenuation (ACFA 2020 continued ...)

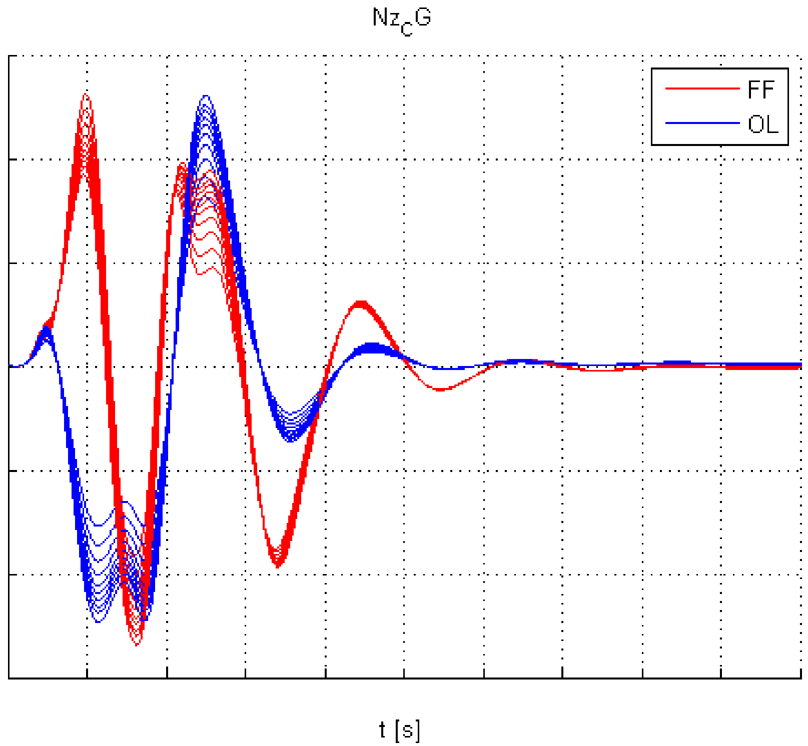



- response from beta disturbance to beta CG (open-loop, FB)

# Control systems: contributions and goals

- measurable disturbance attenuation (ACFA 2020 continued ...)




- alpha (angle-of-attack) signal measured by a probe at the nose
- wind-gusts detected before approaching the wing
- feed-forward controller, with ailerons/flaps as actuators (acting before the disturbance appears)
- two approaches: alpha FIR filtered (Wildschek, Maier), alpha used as trigger (Wildschek, Hanis)
- similar application: active echo cancellation (headphones, car audio, ...)

# Control systems: most common architectures

- systems of interest

  plant, process ... controlled systems

  controller, compensator, control law

  closed-loop system, control loop

- involved signals

  input signal: control input / reference signal

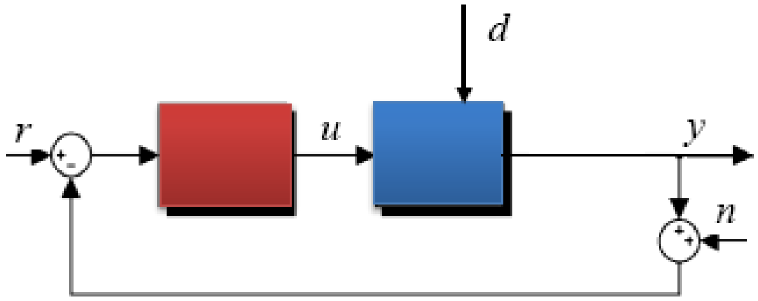
  disturbance: measured / unmeasurable

  output: controlled / measured

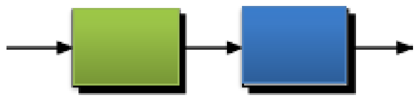
  internal variables (states)

  measurement noise

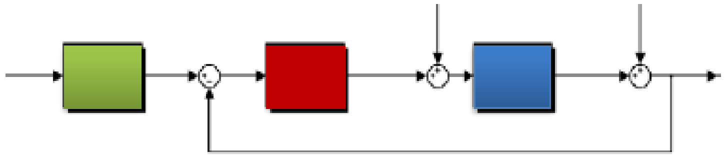
  control error

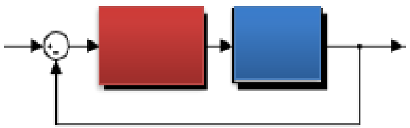


# Control systems: most common architectures

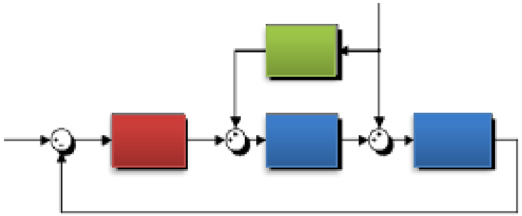


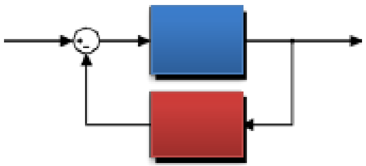
plant



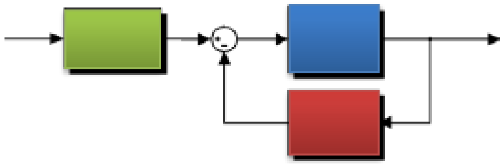


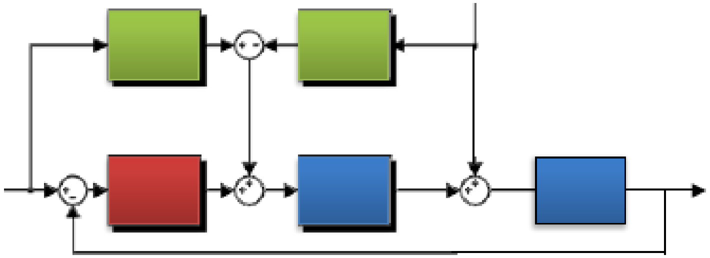
FB controller





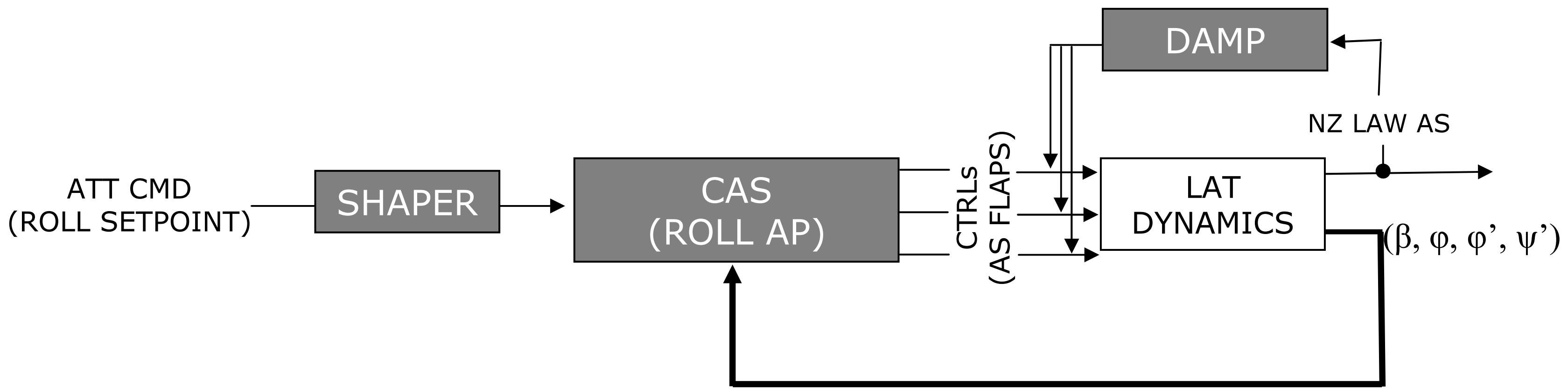
FF controller





# Control systems architectures: ACFA 2020 examples

- reference command tracking




- two-stage design (note advantages due to safety, flight-testing, …):

  - robust roll-AP designed (robust $H_2$ optimal MIMO controller, low order – 6)

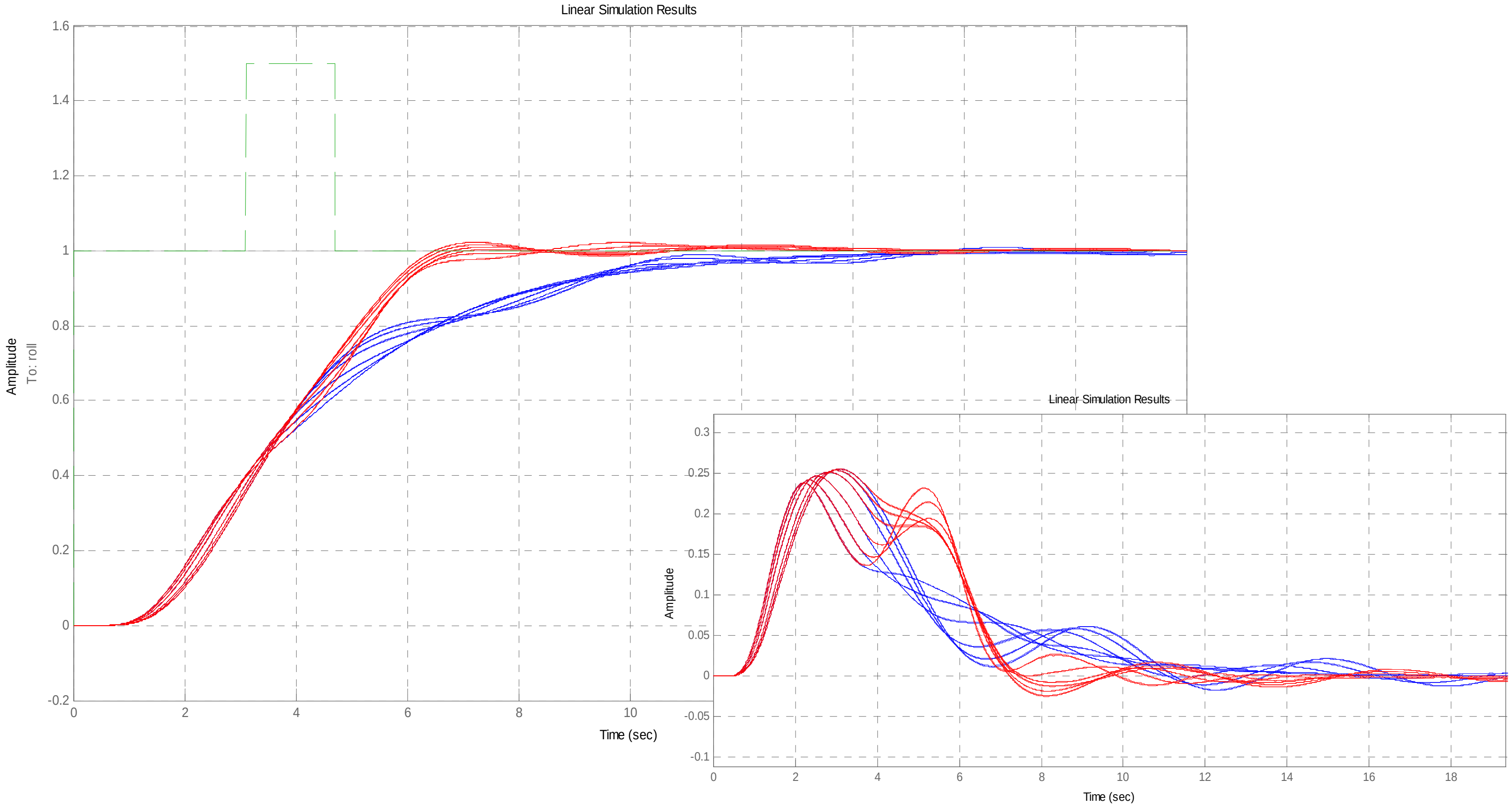
  - $H_\infty$ mixed-sensitivity robust MIMO vibrations damper built upon that

- total order – ~25

- robust for 12 mass cases

# Control systems architectures: ACFA 2020 examples

- reference command tracking

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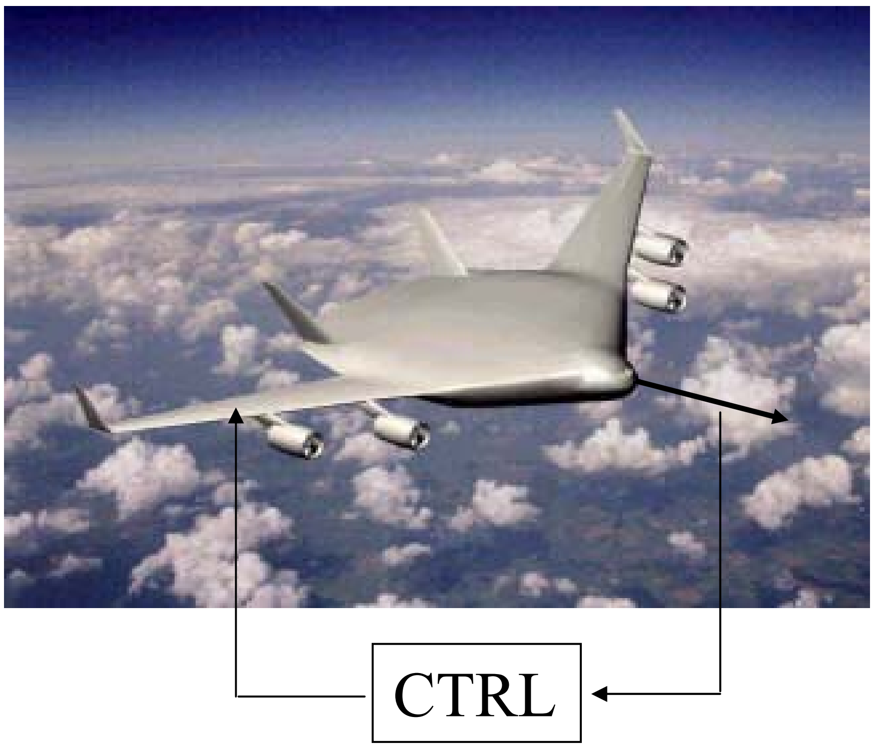
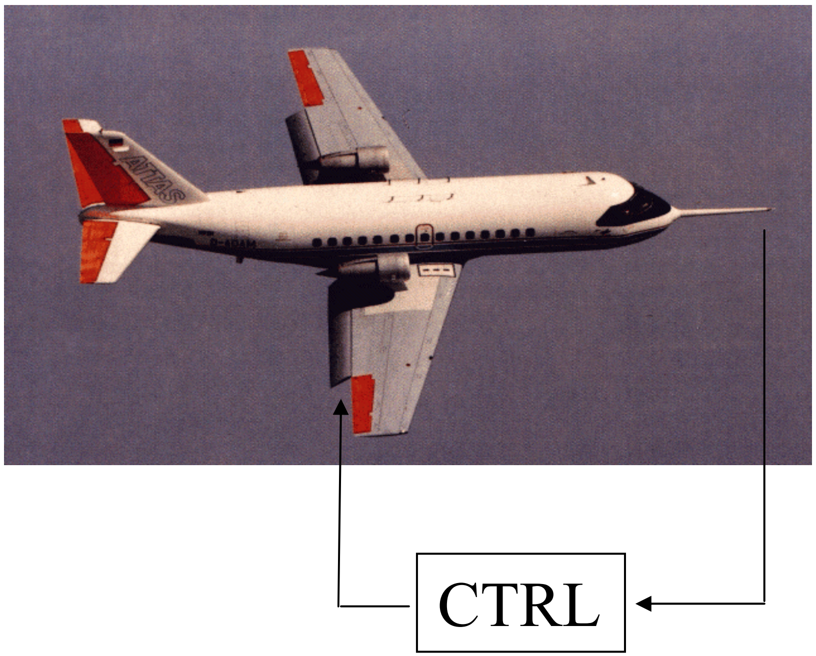
# Control systems architectures: ACFA 2020 examples

- measurable disturbance attenuation

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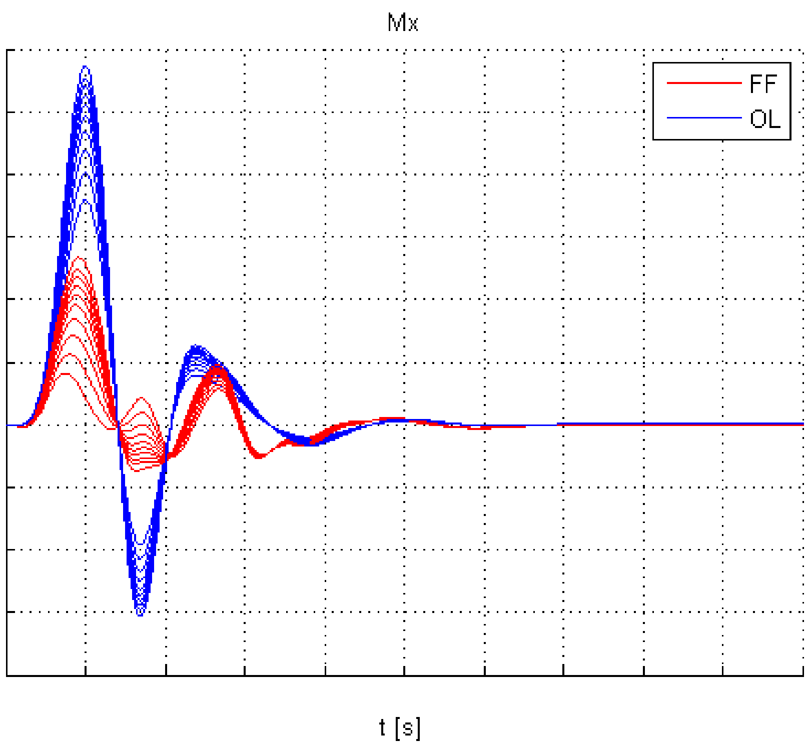


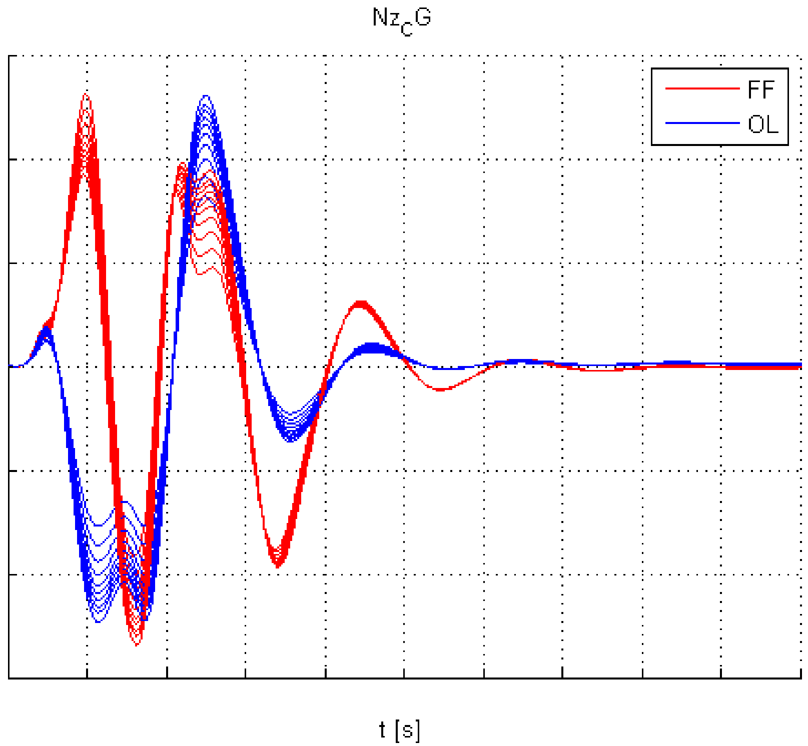


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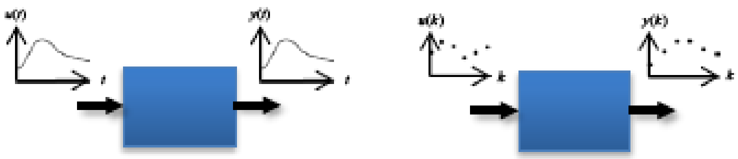
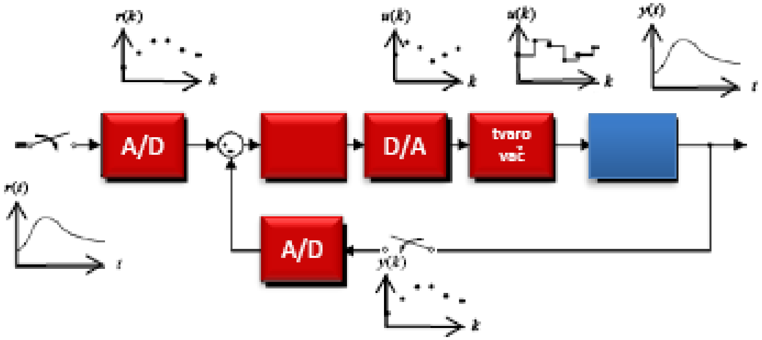
# Control systems further nomenclature

- Continuous/discrete time systems, sampling






- SISO - MIMO



- lumped / distributed parameters (ODE / PDE)
- time varying / time invariant
- linear / nonlinear

# State-space models

- nonlinear

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t)$$
$$\mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t), \mathbf{u}(t), t)$$

- time invariant system
- autonomous system
- static nonlinearity
- equilibrium
- limit cycles

- linear time varying (LTV)

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{D}(t)\mathbf{u}(t)$$

$$\mathbf{x}(t_0^-) = \mathbf{x}_0$$

- homogenity / additivity conditions
- time / frequency domains characteristics

- linear time invariant (LTI)

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$$

$$\mathbf{x}(0^-) = \mathbf{x}_0$$

# I/O models

- nonlinear

$$\mathbf{D}(\mathbf{y}^{(n)}(t),\ldots,\dot{\mathbf{y}}(t),\mathbf{y}(t),t)=\mathbf{N}(\mathbf{u}^{(m)}(t),\ldots,\dot{\mathbf{u}}(t),\mathbf{u}(t),t)$$

$$\mathbf{y}^{(n-1)}(0^-),\ldots,\dot{\mathbf{y}}(0^-),\mathbf{y}(0^-)$$

- linear time varying (LTV)

$$\mathbf{a}_n(t)\mathbf{y}^{(n)}(t)+\cdots+\mathbf{a}_1(t)\dot{\mathbf{y}}+\mathbf{a}_0(t)\mathbf{y}(t)=\mathbf{b}_m(t)\mathbf{u}^{(m)}(t)+\cdots+\mathbf{b}_1(t)\dot{\mathbf{u}}(t)+\mathbf{b}_0(t)\mathbf{u}(t)$$

$$\mathbf{y}^{(n-1)}(0^-),\ldots,\dot{\mathbf{y}}(0^-),\mathbf{y}(0^-)$$

- linear time invariant (LTI)

$$\mathbf{a}_n\mathbf{y}^{(n)}(t)+\cdots+\mathbf{a}_1\dot{\mathbf{y}}(t)+\mathbf{a}_0\mathbf{y}(t)=\mathbf{b}_m\mathbf{u}^{(m)}(t)+\cdots+\mathbf{b}_1\dot{\mathbf{u}}(t)+\mathbf{b}_0\mathbf{u}(t)$$

$$\mathbf{y}^{(n-1)}(0^-),\ldots,\dot{\mathbf{y}}(0^-),\mathbf{y}(0^-)$$

- further / related LTI descriptions: transfer function, Bode plots (frequency characteristics), step / impulse responses
- state-space ⇔ I/O descriptions, realizations, canonical forms

# Linearization

- local linearization only discussed here
- not exact feedback linearization
- nonlinear state-space model

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t)$$

$$\mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t), \mathbf{u}(t), t)$$

- around equilibrium (or, more generally, reference trajectory)

$$\dot{\mathbf{x}}(t) = \dot{\mathbf{x}}_p(t) + \Delta\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}_p(t) + \Delta\mathbf{x}(t), \mathbf{u}_p(t) + \Delta\mathbf{u}(t))$$

$$= \mathbf{f}(\mathbf{x}_p(t), \mathbf{u}_p(t)) + \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{(\mathbf{x}_p, \mathbf{u}_p)} \Delta\mathbf{x}(t) + \left.\frac{\partial \mathbf{f}}{\partial \mathbf{u}}\right|_{(\mathbf{x}_p, \mathbf{u}_p)} \Delta\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{y}_p(t) + \Delta\mathbf{y}(t) = \mathbf{g}(\mathbf{x}_p(t) + \Delta\mathbf{x}(t), \mathbf{u}_p(t) + \Delta\mathbf{u}(t))$$

$$= \mathbf{h}(\mathbf{x}_p(t), \mathbf{u}_p(t)) + \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{(\mathbf{x}_p, \mathbf{u}_p)} \Delta\mathbf{x}(t) + \left.\frac{\partial \mathbf{h}}{\partial \mathbf{u}}\right|_{(\mathbf{x}_p, \mathbf{u}_p)} \Delta\mathbf{u}(t)$$

-higher-order terms omitted

# Linearization

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \qquad \xrightarrow{\mathbf{x}_p(t), \mathbf{u}_p(t)} \qquad \Delta\dot{\mathbf{x}}(t) = \mathbf{A}\Delta\mathbf{x}(t) + \mathbf{B}\Delta\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t), \mathbf{u}(t)) \qquad\qquad \Delta\mathbf{y}(t) = \mathbf{C}\Delta\mathbf{x}(t) + \mathbf{D}\Delta\mathbf{u}(t)$$

$$\mathbf{A} = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{(\mathbf{x}_p, \mathbf{u}_p)}, \mathbf{B} = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{u}}\right|_{(\mathbf{x}_p, \mathbf{u}_p)}, C = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{(\mathbf{x}_p, \mathbf{u}_p)}, D = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{u}}\right|_{(\mathbf{x}_p, \mathbf{u}_p)}$$

$$\mathbf{A} = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{(\mathbf{x}_p, \mathbf{u}_p)} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}_{\mathbf{x}=\mathbf{x}_p, \mathbf{u}=\mathbf{u}_p}, \cdots$$

# Linearization

- nonlinear I/O model (higher-order ODE)

$$\mathbf{D}(\mathbf{y}^{(n)}(t),\ldots,\dot{\mathbf{y}}(t),\mathbf{y}(t),t)=\mathbf{N}(\mathbf{u}^{(m)}(t),\ldots,\dot{\mathbf{u}}(t),\mathbf{u}(t),t)$$

- around equilibrium (or, more generally, reference trajectory)

$$\mathbf{y}(t)=\mathbf{y}_p(t)+\Delta\mathbf{y}(t),\ldots,\mathbf{y}^{(n)}(t)=\mathbf{y}_p^{(n)}(t)+\Delta\mathbf{y}^{(n)}(t),$$
$$\mathbf{u}(t)=\mathbf{u}_p(t)+\Delta\mathbf{u}(t),\ldots,\mathbf{u}^{(m)}(t)=\mathbf{u}_p^{(m)}(t)+\Delta\mathbf{u}^{(m)}(t)$$

$$\mathbf{D}\big|_p+\left.\frac{\partial\mathbf{D}}{\partial\mathbf{y}}\right|_p\Delta\mathbf{y}+\left.\frac{\partial\mathbf{D}}{\partial\dot{\mathbf{y}}}\right|_p\Delta\dot{\mathbf{y}}+\cdots+\left.\frac{\partial\mathbf{D}}{\partial\mathbf{y}^{(n)}}\right|_p\Delta\mathbf{y}^{(n)}+\cdots=\mathbf{N}\big|_p+\left.\frac{\partial\mathbf{N}}{\partial\mathbf{u}}\right|_p\Delta\mathbf{u}+\left.\frac{\partial\mathbf{N}}{\partial\dot{\mathbf{u}}}\right|_p\Delta\dot{\mathbf{u}}+\cdots+\left.\frac{\partial\mathbf{N}}{\partial\mathbf{u}^{(m)}}\right|_p\Delta\mathbf{u}^{(m)}+\cdots$$

$$\left.\frac{\partial\mathbf{D}}{\partial\mathbf{y}}\right|_p\Delta\mathbf{y}+\left.\frac{\partial\mathbf{D}}{\partial\dot{\mathbf{y}}}\right|_p\Delta\dot{\mathbf{y}}+\cdots+\left.\frac{\partial\mathbf{D}}{\partial\mathbf{y}^{(n)}}\right|_p\Delta\mathbf{y}^{(n)}\cong\left.\frac{\partial\mathbf{N}}{\partial\mathbf{u}}\right|_p\Delta\mathbf{u}+\left.\frac{\partial\mathbf{N}}{\partial\dot{\mathbf{u}}}\right|_p\Delta\dot{\mathbf{u}}+\cdots+\left.\frac{\partial\mathbf{N}}{\partial\mathbf{u}^{(m)}}\right|_p\Delta\mathbf{u}^{(m)}$$

$$\mathbf{a}_0\Delta\mathbf{y}+\mathbf{a}_1\Delta\dot{\mathbf{y}}+\cdots+\mathbf{a}_n\Delta\mathbf{y}^{(n)}=\mathbf{b}_0\Delta\mathbf{u}+\mathbf{b}_1\Delta\dot{\mathbf{u}}+\cdots+\mathbf{b}_m\Delta\mathbf{u}^{(m)}$$

# Discrete-time models

- nonlinear SS models

$$\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k), \mathbf{u}(k), k) \qquad \mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k), \mathbf{u}(k))$$

$$\mathbf{y}(k) = \mathbf{h}(\mathbf{x}(k), \mathbf{u}(k), k) \qquad \mathbf{y}(k) = \mathbf{h}(\mathbf{x}(k), \mathbf{u}(k))$$

- linear SS models

$$\mathbf{x}(k+1) = \mathbf{A}(k)\mathbf{x}(k) + \mathbf{B}(k)\mathbf{u}(k) \qquad \mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k)$$

$$\mathbf{y}(k) = \mathbf{C}(k)\mathbf{x}(k) + \mathbf{D}(k)\mathbf{u}(k) \qquad \mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k)$$

- initial conditions

$$\mathbf{x}(k_0) = \mathbf{x}_0 \qquad \mathbf{x}(0) = \mathbf{x}_0$$

- equilibrium

$$\mathbf{u}_e(k) = \mathbf{u}_e,\ \mathbf{x}_e(k) = \mathbf{x}_e \Rightarrow \mathbf{x}_e = \mathbf{f}(\mathbf{x}_e, \mathbf{u}_e)$$

# Discrete-time models

- nonlinear IO model

$$\mathbf{D}(\mathbf{y}(k+n),\ldots,\mathbf{y}(k+1),\mathbf{y}(k),k)=\mathbf{N}(\mathbf{u}(k+m),\ldots,\mathbf{u}(k+1),\mathbf{u}(k),k)$$

- linear IO model

$$\mathbf{a}_n(k)\mathbf{y}(k+n)+\cdots+\mathbf{a}_1(k)\mathbf{y}(k+1)+\mathbf{a}_0(k)\mathbf{y}(k)$$
$$=\mathbf{b}_m(k)\mathbf{u}(k+m)+\cdots+\mathbf{b}_1(k)\mathbf{u}(k+1)+\mathbf{b}_0(k)\mathbf{u}(k)$$

- LTI IO model (relate to transfer function, Z-transform)

$$\mathbf{a}_n\mathbf{y}(k+n)+\cdots+\mathbf{a}_1\mathbf{y}(k+1)+\mathbf{a}_0\mathbf{y}(k)$$
$$=\mathbf{b}_m\mathbf{u}(k+m)+\cdots+\mathbf{b}_1\mathbf{u}(k+1)+\mathbf{b}_0\mathbf{u}(k)$$

- initial conditions, equilibrium
- linearization

$$\Delta\mathbf{x}(k+1)\cong\left.\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right|_{(\mathbf{x}_p,\mathbf{u}_p)}\Delta\mathbf{x}(k)+\left.\frac{\partial\mathbf{f}}{\partial\mathbf{u}}\right|_{(\mathbf{x}_p,\mathbf{u}_p)}\Delta\mathbf{u}(k)$$
$$\Delta\mathbf{y}(k)\cong\left.\frac{\partial\mathbf{h}}{\partial\mathbf{x}}\right|_{(\mathbf{x}_p,\mathbf{u}_p)}\Delta\mathbf{x}(k)+\left.\frac{\partial\mathbf{h}}{\partial\mathbf{u}}\right|_{(\mathbf{x}_p,\mathbf{u}_p)}\Delta\mathbf{u}(k)$$