Exercises for lectures
6 – Interconnection and structures

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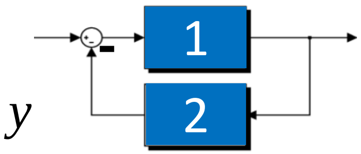
Derivation of the general approach



$$u_1 = u - y_2, y_1 = u_2 = y$$

$$y = y_1 = \mathbf{C}_1\mathbf{x}_1 + \mathbf{D}_1 u_1 = \mathbf{C}_1\mathbf{x}_1 + \mathbf{D}_1\left(u - y_2\right)$$
$$= \mathbf{C}_1\mathbf{x}_1 + \mathbf{D}_1 u - \mathbf{D}_1\left(\mathbf{C}_2\mathbf{x}_2 + \mathbf{D}_2 y\right)$$

$$\left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right) y = \mathbf{C}_1\mathbf{x}_1 + \mathbf{D}_1\mathbf{C}_2\mathbf{x}_2 + \mathbf{D}_1 u$$

$$y = \left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right)^{-1}\mathbf{C}_1\mathbf{x}_1 + \left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right)^{-1}\mathbf{D}_1\mathbf{C}_2\mathbf{x}_2$$
$$+ \left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right)^{-1}\mathbf{D}_1 u$$

$$\dot{\mathbf{x}}_1 = \mathbf{A}_1\mathbf{x}_1 + \mathbf{B}_1 u_1 = \mathbf{A}_1\mathbf{x}_1 + \mathbf{B}_1\left(u - y_2\right)$$
$$= \mathbf{A}_1\mathbf{x}_1 + \mathbf{B}_1 u - \mathbf{B}_1\left(\mathbf{C}_2\mathbf{x}_2 + \mathbf{D}_2 y\right)$$
$$= \mathbf{A}_1\mathbf{x}_1 + \mathbf{B}_1 u - \mathbf{B}_1\mathbf{C}_2\mathbf{x}_2 - \mathbf{B}_1\mathbf{D}_2 y$$
$$= \mathbf{A}_1\mathbf{x}_1 - \mathbf{B}_1\mathbf{D}_2\left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right)^{-1}\mathbf{C}_1\mathbf{x}_1 - \mathbf{B}_1\mathbf{C}_2\mathbf{x}_2$$
$$-\mathbf{B}_1\mathbf{D}_2\left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right)^{-1}\mathbf{D}_1\mathbf{C}_2\mathbf{x}_2 + \mathbf{B}_1 u - \mathbf{B}_1\mathbf{D}_2\left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right)^{-1}\mathbf{D}_1 u$$

$$\dot{\mathbf{x}}_2 = \mathbf{A}_2\mathbf{x}_2 + \mathbf{B}_2 u_2 = \mathbf{A}_2\mathbf{x}_2 + \mathbf{B}_2 y =$$
$$\mathbf{A}_2\mathbf{x}_2 + \mathbf{B}_2\left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right)^{-1}\mathbf{C}_1\mathbf{x}_1 + \mathbf{B}_2\left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right)^{-1}\mathbf{D}_1\mathbf{C}_2\mathbf{x}_2$$
$$+\mathbf{B}_2\left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right)^{-1}\mathbf{D}_1 u$$

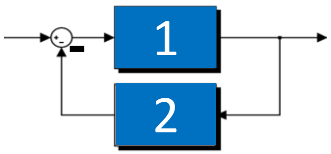
$$\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{A}_1 - \mathbf{B}_1\mathbf{D}_2\left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right)^{-1}\mathbf{C}_1 & -\mathbf{B}_1\mathbf{C}_2 - \mathbf{B}_1\mathbf{D}_2\left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right)^{-1}\mathbf{D}_1\mathbf{C}_2 \\ \mathbf{B}_2\left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right)^{-1}\mathbf{C}_1 & \mathbf{A}_2 + \mathbf{B}_2\left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right)^{-1}\mathbf{D}_1\mathbf{C}_1 \end{bmatrix}\mathbf{x}$$
$$+ \begin{bmatrix} \mathbf{B}_1 - \mathbf{B}_1\mathbf{D}_2\left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right)^{-1}\mathbf{D}_1 \\ \mathbf{B}_2\left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right)^{-1}\mathbf{D}_1 \end{bmatrix} u$$

$$y = \begin{bmatrix} \left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right)^{-1}\mathbf{C}_1 & \left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right)^{-1}\mathbf{D}_1\mathbf{C}_2 \end{bmatrix}\mathbf{x} + \left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right)^{-1}\mathbf{D}_1 u$$

$$\det\left(\mathbf{I} + \mathbf{D}_1\mathbf{D}_2\right) \neq 0$$

$$\mathbf{I} + \mathbf{D}_1\mathbf{D}_2 =$$
$$= \lim_{s\to\infty}\left(1 + f_1(s) f_2(s)\right)$$

Proper systems



$$\big(1+F_1(s)F_2(s)\big)=0$$ the final system doesn't have a transfer function !

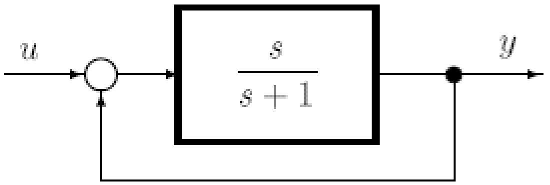
$$\det\left(\mathbf{I}+\mathbf{D}_1\mathbf{D}_2\right)\neq 0$$ the final system doesn't have a state representation !

because $\mathbf{I}+\mathbf{D}_1\mathbf{D}_2=\lim_{s\to\infty}\big(1+F_1(s)F_2(s)\big)$:

$$\det\left(\mathbf{I}+\mathbf{D}_1\mathbf{D}_2\right)\neq 0 \quad \begin{matrix}\Rightarrow \\ \not\Leftarrow\end{matrix} \quad 1+F_1(s)F_2(s)\neq 0$$

Example – improper system

$$F(s)=\frac{\frac{s}{s+1}}{1-\frac{s}{s+1}}=s$$



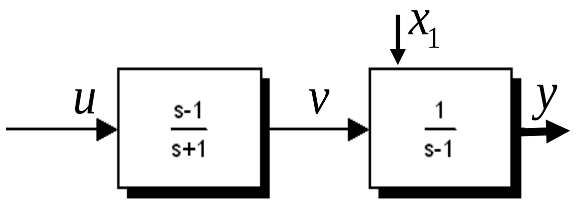
- The final system is improper, doesn't have a state representation Despite the fact that the subsystems are proper and they have state representations
- Connection of proper systems led to improper final system

Clarification of weird examples

Example with input zero:

- The total characteristic polynomial is

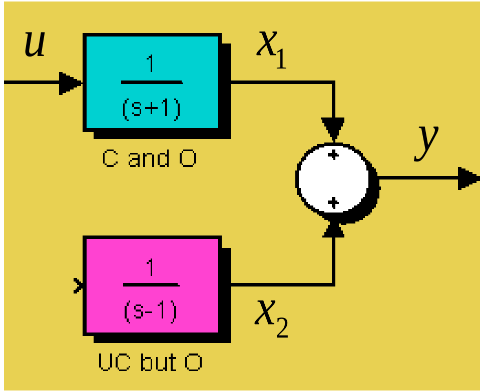
$$c(s) = (s+1)(s-1)$$



Example „branch without input“

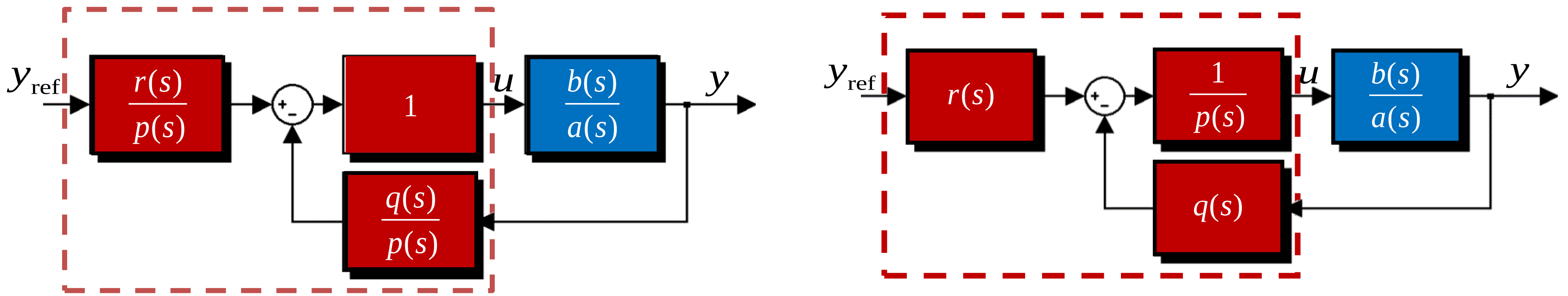
- The total characteristic polynomial is

$$c(s) = (s+1)(s-1)$$



Difference

- What is the difference between these two systems?



- Transient from reference to output seems to be the same, but try characteristic polynomial :
- Assuming that the subsystems don't have a hidden mode, ei. $a(s), p(s)$ are characteristic polynomials in their blocks
- The left system has characteristic polynomial $$c(s) = \big(a(s)p(s) + b(s)q(s)\big)\,p(s)$$
- The right systém has $$c(s) = a(s)p(s) + b(s)q(s)$$
- What makes the difference?