Exercises for lectures
6 – Interconnection and structures

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Derivation of the general approach

\[ y = y_1 = C_1 x_1 + D_1 u_1 = C_1 x_1 + D_1 (u - y_2) \]
\[ = C_1 x_1 + D_1 u - D_1 (C_2 x_2 + D_2 y) \]
\[ (I + D_1 D_2) y = C_1 x_1 + D_1 C_2 x_2 + D_1 u \]
\[ y = (I + D_1 D_2)^{-1} C_1 x_1 + (I + D_1 D_2)^{-1} D_1 C_2 x_2 + (I + D_1 D_2)^{-1} D_1 u \]

\[ \dot{x} = \begin{bmatrix}
A_1 - B_1 D_2 (I + D_1 D_2)^{-1} C_1 & -B_1 C_2 - B_1 D_2 (I + D_1 D_2)^{-1} D_1 C_2 \\
B_2 (I + D_1 D_2)^{-1} C_1 & A_2 + B_2 (I + D_1 D_2)^{-1} D_1 C_1 \\
\end{bmatrix} x + \begin{bmatrix}
B_1 - B_1 D_2 (I + D_1 D_2)^{-1} D_1 \\
B_2 (I + D_1 D_2)^{-1} D_1 \\
\end{bmatrix} u \]
\[ y = \begin{bmatrix}
(I + D_1 D_2)^{-1} C_1 & (I + D_1 D_2)^{-1} D_1 C_2 \\
\end{bmatrix} x + (I + D_1 D_2)^{-1} D_1 u \]

\[ u_1 = u - y_2, \ y_1 = u_2 = y \]
\[ \dot{x}_1 = A_1 x_1 + B_1 u_1 = A_1 x_1 + B_1 (u - y_2) \]
\[ = A_1 x_1 + B_1 u - B_1 (C_2 x_2 + D_2 y) \]
\[ = A_1 x_1 + B_1 u - B_1 C_2 x_2 - B_1 D_2 y \]
\[ \dot{x}_2 = A_2 x_2 + B_2 u_2 = A_2 x_2 + B_2 y = A_2 x_2 + B_2 (I + D_1 D_2)^{-1} C_1 x_1 + B_2 (I + D_1 D_2)^{-1} D_1 C_2 x_2 + B_2 (I + D_1 D_2)^{-1} D_1 u \]

\[ \text{det}(I + D_1 D_2) \neq 0 \]
\[ I + D_1 D_2 = \lim_{s \to \infty} (1 + f_1(s)f_2(s)) \]
Proper systems

\[
(1 + F_1(s)F_2(s)) = 0 \quad \text{the final system doesn't have a transfer function!}
\]

\[
\det(I + D_1D_2) \neq 0 \quad \text{the final system doesn't have a state representation!}
\]

\[
I + D_1D_2 = \lim_{s \to \infty} (1 + F_1(s)F_2(s))
\]

because

\[
\det(I + D_1D_2) \neq 0 \quad \Rightarrow \quad 1 + F_1(s)F_2(s) \neq 0
\]
Example – improper system

\[ F(s) = \frac{s}{s+1} = s \]

- The final system is improper, doesn't have a state representation. Despite the fact that the subsystems are proper and they have state representations.
- Connection of proper systems led to improper final system.
Example with input zero:
• The total characteristic polynomial is
  \[ c(s) = (s + 1)(s - 1) \]

Example „branch without input“
• The total characteristic polynomial is
  \[ c(s) = (s + 1)(s - 1) \]
• What is the difference between these two systems?

• Transient from reference to output seems to be the same, but try characteristic polynomial:

• Assuming that the subsystems don't have a hidden mode, ei. \( a(s), p(s) \) are characteristic polynomials in their blocks

• The left system has characteristic polynomial

\[
c(s) = \left( a(s)p(s) + b(s)q(s) \right) p(s)
\]

• The right system has

\[
c(s) = a(s)p(s) + b(s)q(s)
\]

• What makes the difference?