Exercises for lectures
11 - Controllers

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Plants with oscillating modes

- Some important applications in conventional industrial processes have a strongly oscillating modes:
  - Flexible robot arm
  - Disc drive
  - AMF (Atomic Force Microscope)
  - MEMS (Micro-Electro-Mechanical Systems)
  - Flexible structure in the cosmos
  - Combustion system
- These systems are hard to control, particularly when the damping is very small and the system resonates.
- It is almost impossible to control with PI – does not increase phase margin, the closed loop is even less damped.
- PI regulator mustn't excite oscillatory modes, therefore the resulting reaction is very slow.
- D action helps significantly.

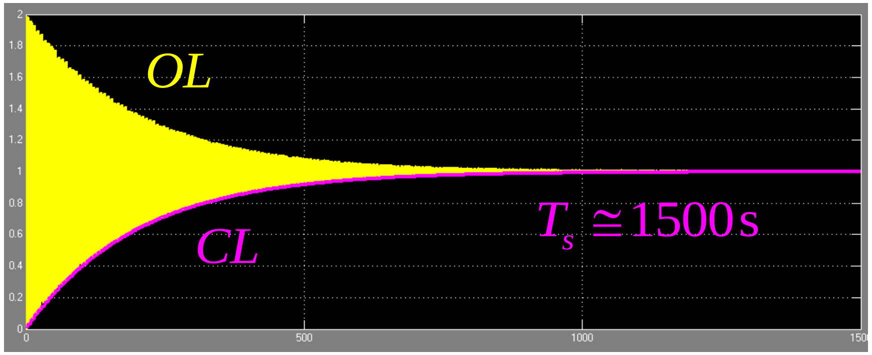
Example: Weekly damped oscillatory plant

AH_3_5_Oscil.mdl

- Oscillating system with a small damping ratio

$$G(s)=\frac{a^2}{s^2+2\varsigma as+a^2}$$

$$\varsigma=0.005$$

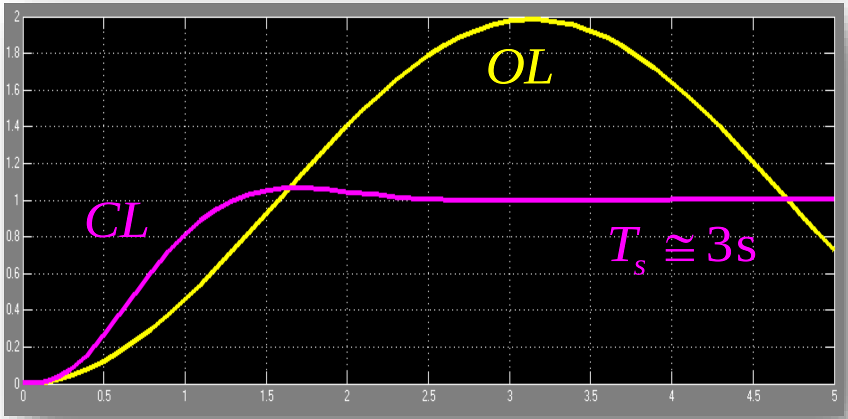


- I regulator (P helps only little)

$$C(s)=\frac{0.005}{s}$$

- PID regulator

$$C(s)=17+\frac{27}{s}+5.99s$$



- Even better $b=0$, then step does not excite high frequencies.

Example: Higher order plant

AH_Ex3_3_HiOr.mdl

- System of 3rd order

$$G(s)=\frac{1}{(s+1)^3}$$

- PID regulator

$$C(s)=3.5\left(1+\frac{1}{2.0s}+0.6s\right)$$

- TDF regulator of 3rd order

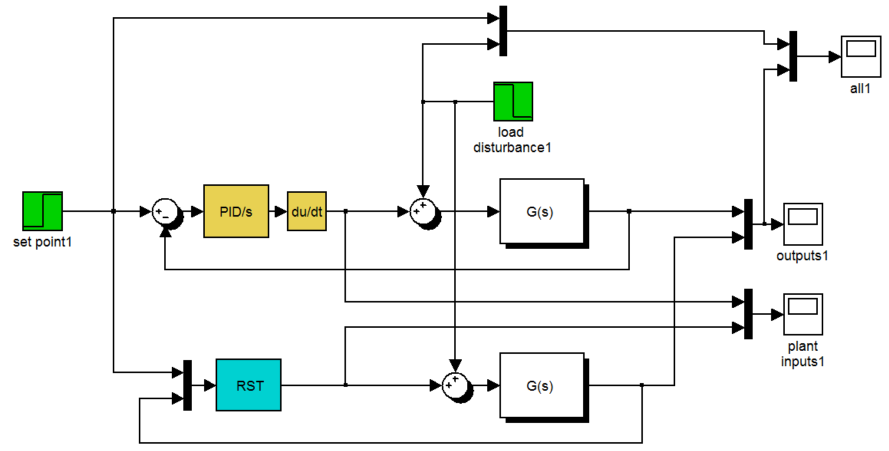
$$R(s)u=-S(s)y+T(s)y_{sp}$$

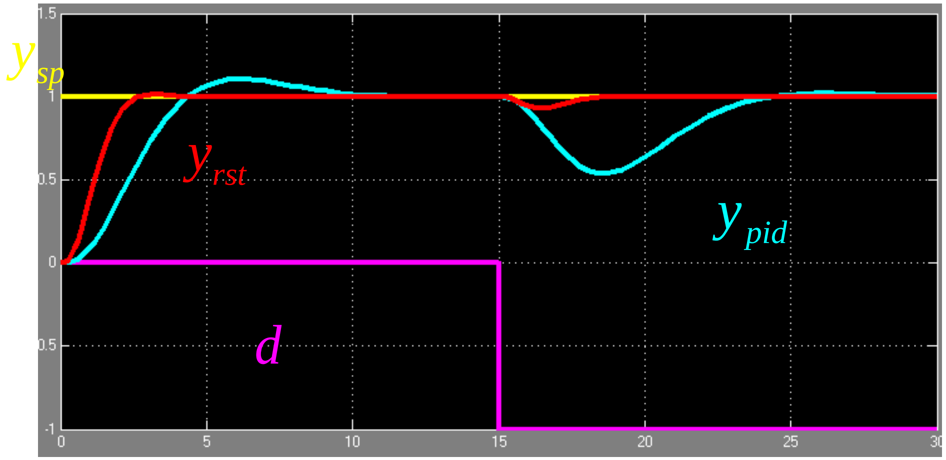
$$R(s)=s(s^2+11.5s+57.5)$$

$$S(s)=144s^3+575s^2+870s+512$$

$$T(s)=8s^3+77s^2+309s+512$$

is better than PID





Example: System with transport delay

AH_Ex3_4_TD.mdl

- System with a large transport delay

$$G(s) = \frac{1}{1+2s} e^{-4s}$$

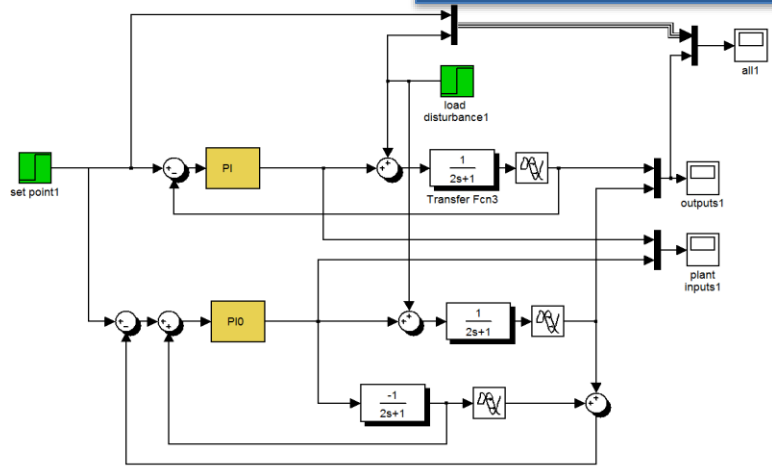
- PI regulator (D action doesn't help)

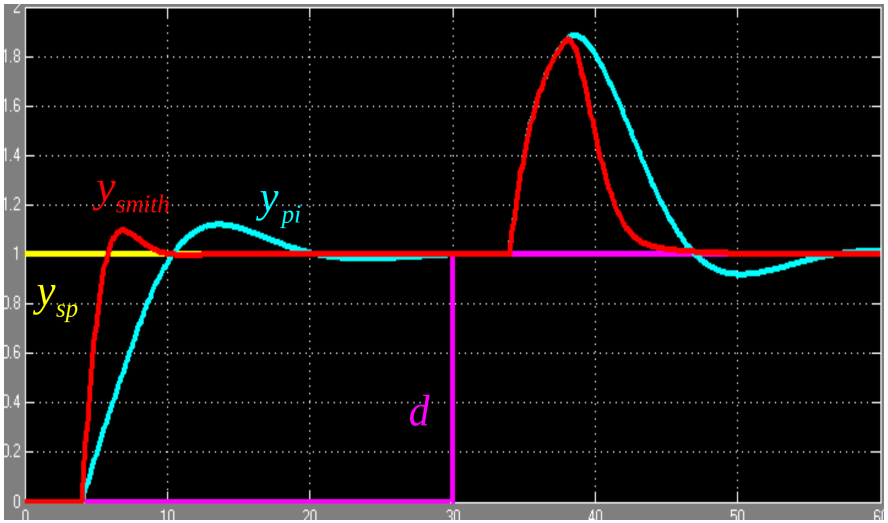
$$C(s) = 0.4\left(1 + \frac{1}{2.5s}\right)$$

- Smith predictor
  with a PI regulator

$$C_0(s) = 1.8\left(1 + \frac{1}{0.9s}\right)$$

is better in comparison to PID:
It has better responses to the step of input and the step of disturbance.





Fast response – pulse input

- Larger the action ➡ faster the response – limited in practice
- The fasters response gives a pulse input „bang-bang“ $u(t) \in [u_{\min}, u_{\max}]$
- The input signal can be calculated (time optimal control) – is non-linear

Example

- System

$$P(s) = \frac{1}{(s+1)^4} \qquad u_{\min} = -4 \qquad u_{\max} = 4$$

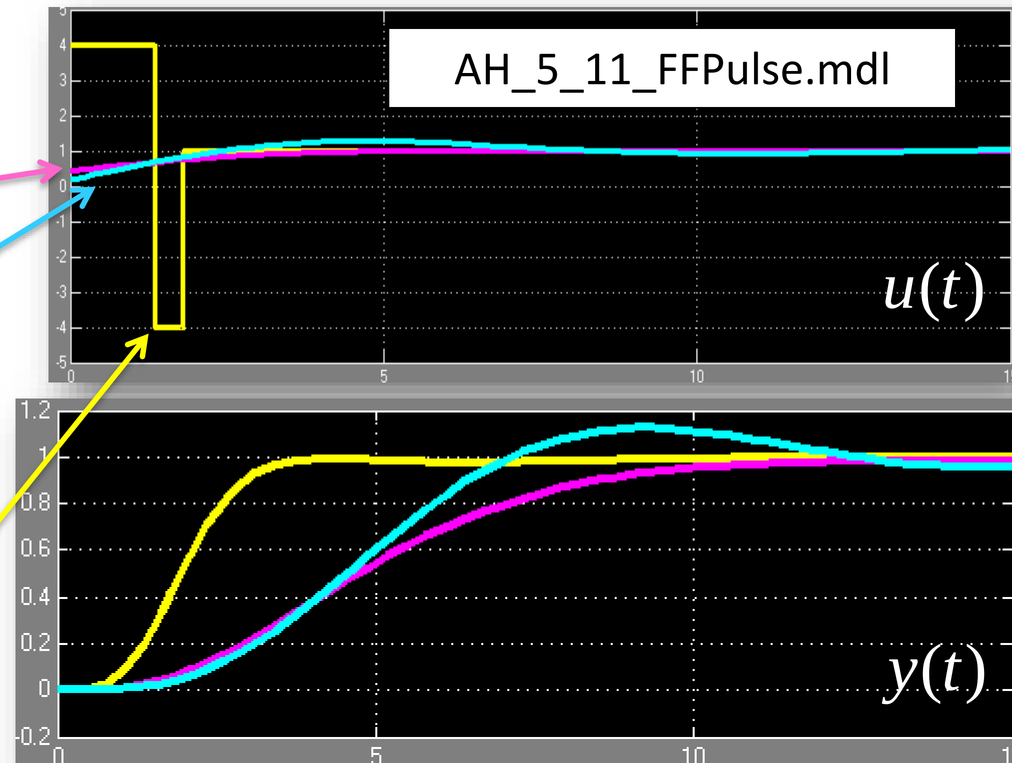
- PI regulator

$$K = 0.43, T_i = 2.25$$
$$b = 1, (M_S = 1.4)$$

- PI regulator

- $$K = 0.78, T_i = 2.05$$
$$b = 0.23, (M_S = 2.0)$$

- Pulse FF



Fast response – limited response speed

- Other practical limit: the speed of action signal
- Combined limitations: bounded value and speed of the action signal
- Is also non-linear

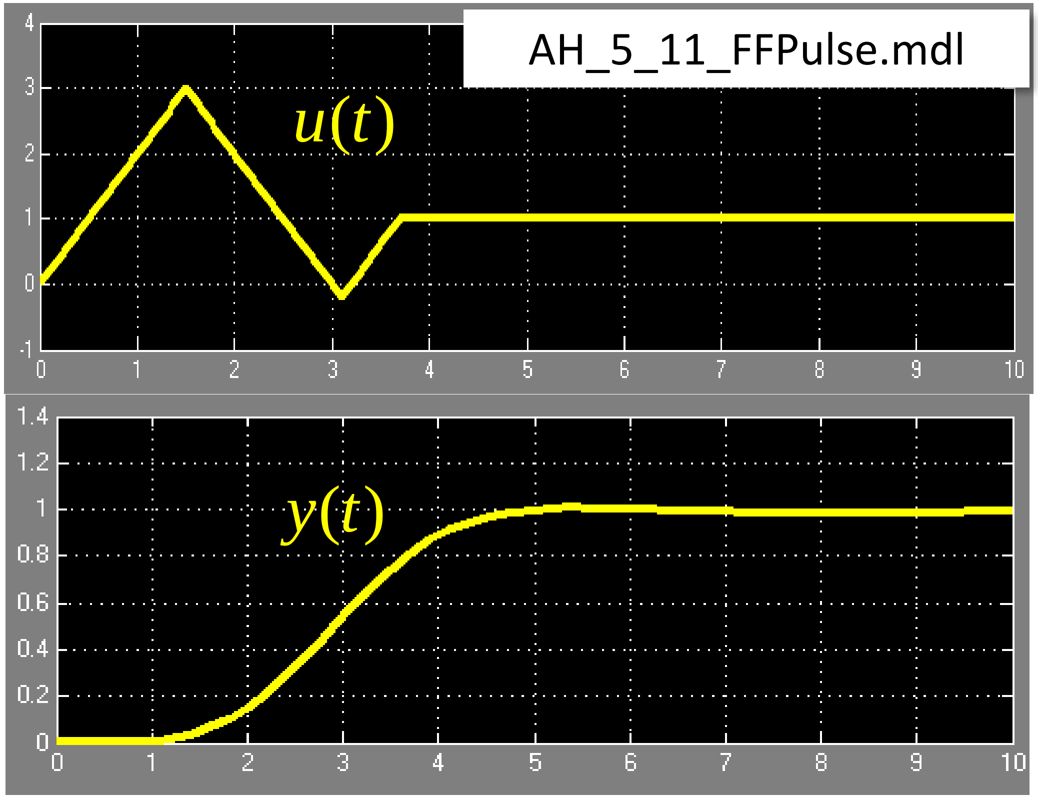
Example

- System (same as previously)

$$P(s) = \frac{1}{(s+1)^4}$$

- but it must hold

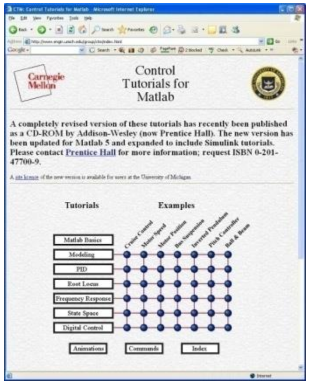
$$\left|\frac{du}{dt}\right| < \text{konst}$$



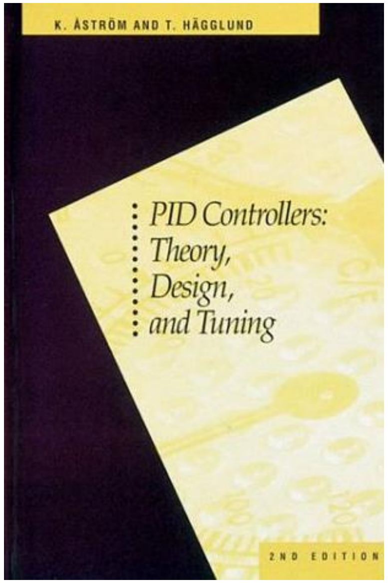
Some fun with P-I-D and other regulators

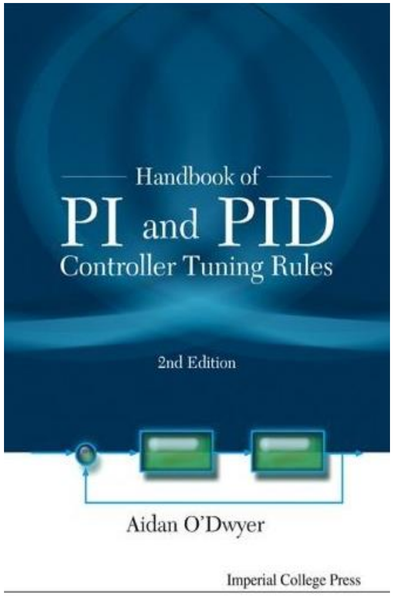
- The Internet is full of interesting pages about PID controllers.
- For example:

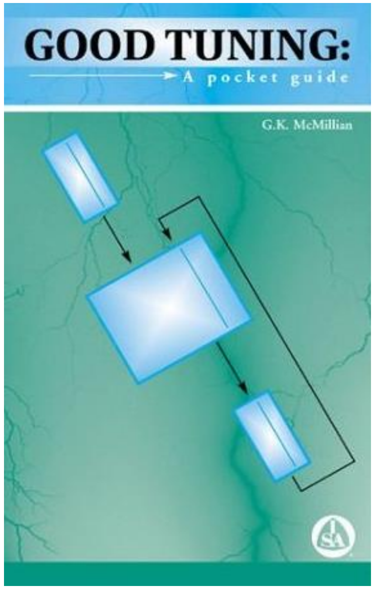
www.engin.umich.edu/group/ctm/index.html

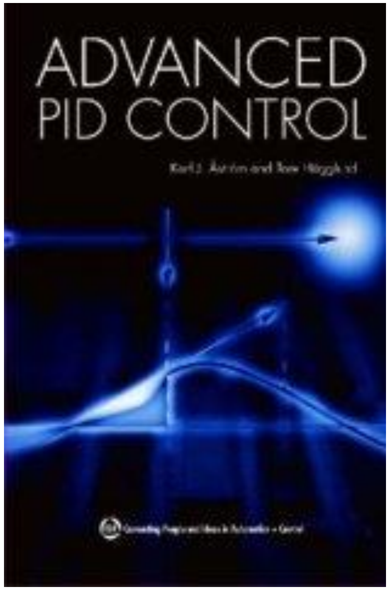


- Entire books address the tuning of PID controllers:









Example: Counter-intuitive behavior

- The usual rule for manual tuning says, that if we reduce K, we increase the stability and suppress oscillations (we increase damping).
- It usually holds, but not always.
- Consider system $G(s)=\frac{1}{s}$ with a PI regulator $C(s)=K_P\left(1+\frac{1}{T_i s}\right)$
- Characteristic polynomial of the closed loop is

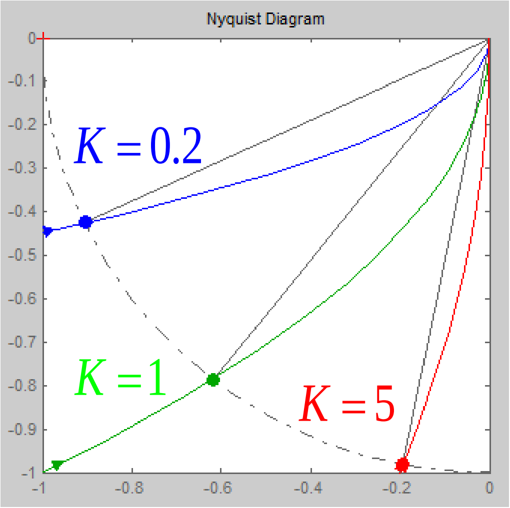
$$p_{cl}(s)=T_i s^2+K_P T_i s+K \Rightarrow s^2+K_P s+\frac{K}{T_i}$$

- In comparison with the general polynomial for 2nd order system $s^2+2\zeta\omega_n s+\omega_n^2$
- we calculate damping ratio

$$\varsigma=\frac{\sqrt{K_P T_i}}{2}$$

- that obviously depends on K_P - the opposite, what the rule says.

Against intuition: *PM* rises with K_P



Example: 2[nd] order system and PID regulator

- We use the pole placement method, when other methods does not work.
- System with an unstable zero and weekly damped oscillatory modes
- This example can not be solved by other (traditional) methods. $\frac{b(s)}{a(s)} = \frac{1-s}{s^2+1}$
  (see Åström, Hägglund: Advanced PID Control, s 180)
- Lets choose $c(s) = s^3 + 2s^2 + 2s + 1 = (s+1)(s+0.5+j0.866)(s+0.5-j0.866)$
- Then we assemble the system and solve it (PolTbx)

```
>> c=s^3+2*s^2+2*s+1,a=s^2+1,b=1-s
c = 1 + 2s + 2s^2 + s^3
a = 1 + s^2
b = 1 - s
>> [x,y]=axbyc(a*s,b,c)
x =  3.0000
y =  1 + 2s^2
```

$$\frac{q(s)}{p(s)} = \frac{k_D s^2 + k_P s + k_I}{s} = \frac{2s^2+1}{3s}$$

$$k_P = 0$$
$$k_I = 1/3$$
$$k_D = 2/3$$

Pole placement example of one pole

$$G(s)=\frac{b(s)}{a(s)}=\frac{1}{(s+1)^2} \qquad D_C(s)=\frac{k_I}{s}$$

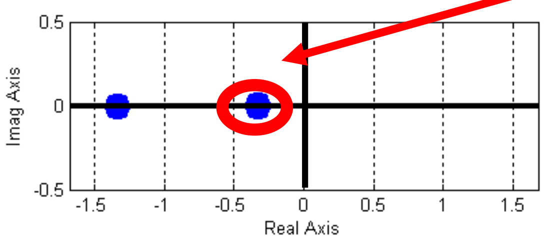
$$k_I=\frac{h}{G(-h)}=h(1-h)^2=h^3-2h^2+h$$

- We choose $s=-h, h>0$, then the pole will be determined by this constant
- We choose for example $h=1/3$,i.e. a pole in $s=-1/3$ then we need a constant
- I with a transfer function $D_C(s)=\frac{4/27}{s}$ $\quad k_I=4/27$

  gives a CL characteristic polynomial

$$c(s) \;=\; 0.15 \;+\; s \;+\; 2s^2 \;+\; s^3$$

- Its roots, i.e. CL poles are




```
>> format rat
>> P=1/(s+1)^2;
>> h=1/3,kI=h/value(P,-h)
h = 1/3      kI = 4/27
>> D=kI/s
D =  0.15 / s
>> c=P.den*D.den+P.num*D.num
c = 0.15 + s + 2s^2 + s^3
>> roots(c)
ans = -4/3
      -1/3 +  1/297399692i
      -1/3 -  1/297399692i
```

- One of poles (actually a pole pair) was placed into required position and it is „dominant“.

Useful tricks: Filtration of a derivative

- Ideal derivative has a very high gain at high frequencies

$$y = \underbrace{\sin t}_{\text{signal}} + \underbrace{a \sin \omega t}_{\text{noise}} \quad \Rightarrow \quad \frac{dy}{dt} = \cos t + a\omega \cos \omega t$$

noise ration : signal $= a \quad \Rightarrow \quad = a\omega$

- Therefore we usually filtrate it: instead of $D = KT_d s$ we use

$$D = \frac{KT_d s}{1 + sT_d / N} \quad \begin{cases} \omega \downarrow & D \cong KT_d s \\ \omega \uparrow & D \cong KN, N \in [2, 20] \end{cases}$$

- Alternatively, we filtrate not only D, but all regulator components

$$\tilde{C}(s) = C(s) C_f(s) = K\left(1 + \frac{1}{sT_i} + sT_d\right) \frac{1}{1 + sT_f + (sT_f)^2 / 2}$$

2nd order filter with a damping ratio $\varsigma = 1/\sqrt{2}$

and a time constant $T_f = T_i / N$ for PI and $T_f = T_d / N$ for PID

high frequency roll-off ↓

$$\lim_{\omega \to \infty} \tilde{C}(j\omega) = 0$$

Useful tricks : Set-point weighting

- Often, a more flexible structure is used

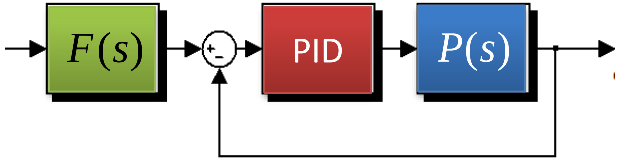
$$e = y_{sp} - y$$

The integrated value remains in the integral term because of the zero regulation error!

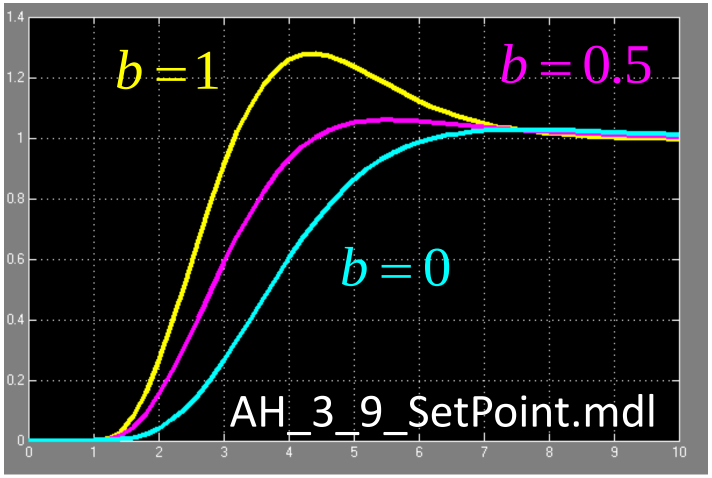
$$u(t) = K\left(e_p(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d \frac{de_d(t)}{dt} \right)$$

$$e_p = by_{sp} - y, \quad e_d = cy_{sp} - y$$

- We further fine tune the regulator by changing the weights.
  For example $b = 0$ slows the response to change, but reduces the overshoot.
  It is equivalent to a structure with a standard PID a feed-forward F



$$F(s) = \frac{cT_iT_ds^2 + bsT_i + 1}{T_iT_ds^2 + sT_i + 1}$$

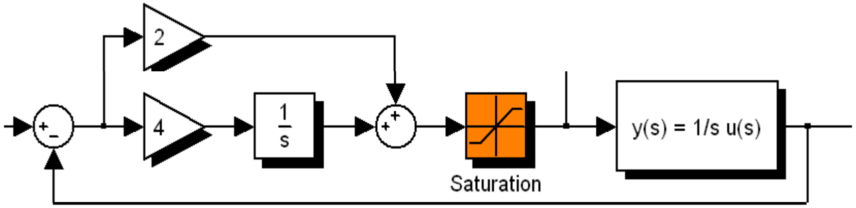


- The choice of the weights b, c influences zeros of the final transfer function.

Windup

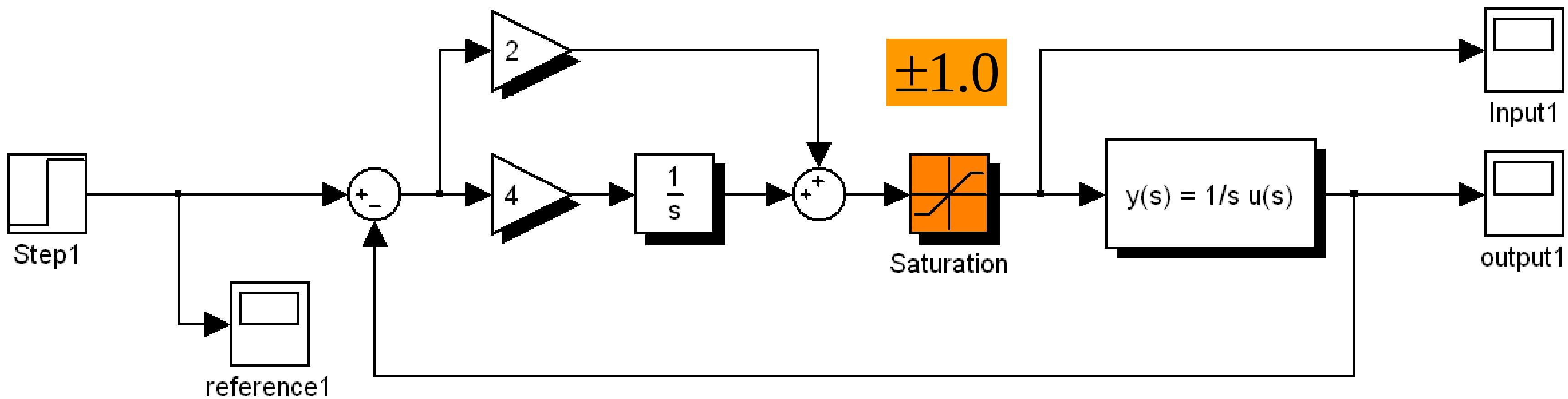
Actuator saturation

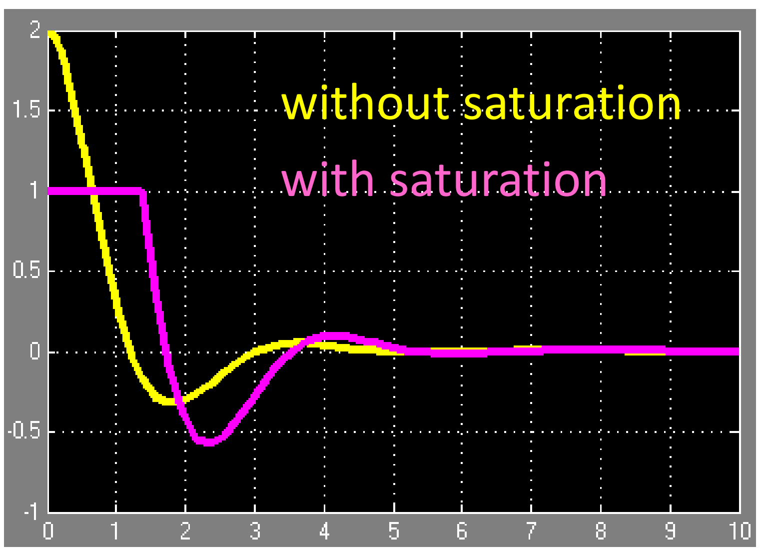
- Every real actuator has a limited range.
  - A valve may be at most "totally open“ and at least "fully closed“.
  - Ailerons can not deflect beyond a certain angle from the nominal position
  - Electronic amplifiers can produce only a maximum final voltage
- When the action is saturated
  - Control signal does not increase/decrease
    and the loop is essentially opened
    - The output of the regulator integral term permanently increases its value, but it is not useless
  - When the sign changes, the control deviation begins to decline, but it takes long before it gets below the level of saturation.

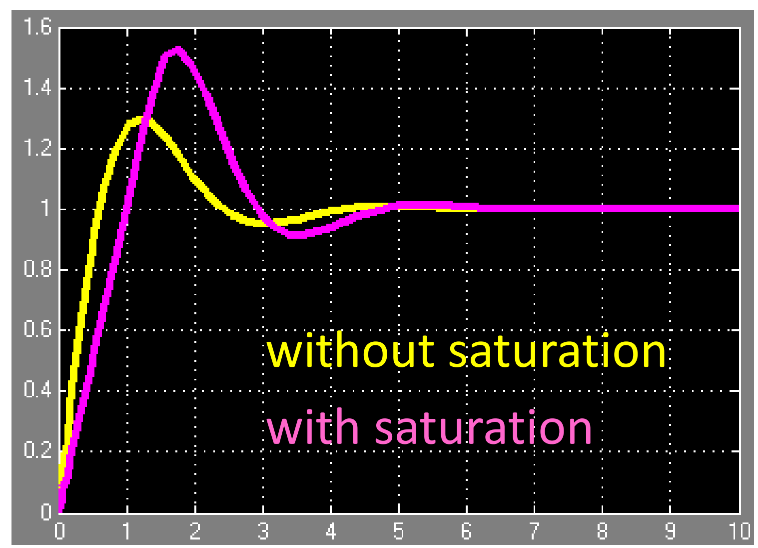


- The result is a big overshoot and wrong response to step.
- The integral term is an unstable element in the open loop, and must be independently stabilized.

Anti-Windup


2
4
1
s
±1.0
Step1
reference1
Saturation
y(s) = 1/s u(s)
Input1
output1


without saturation
with saturation
2
1.5
1
0.5
0
-0.5
-1
0
1
2
3
4
5
6
7
8
9
10


without saturation
with saturation
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8
9
10

Anti-Windup

- The anti-windup circuit offers a solution to this problem. It "turns off" integral action as soon as its output is saturated.
- This reduces the overshoot and changes the step response.

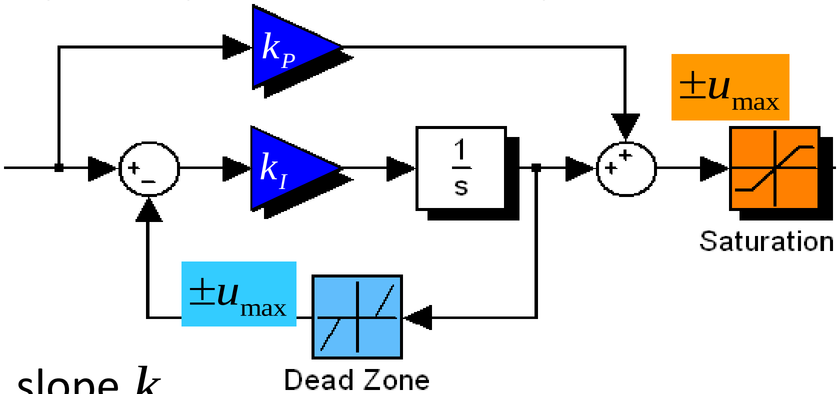
- In terms of stability, the saturation non-linearity causes temporary disconnection of the loop.
- The purpose of the anti-windup circuit is to stabilize the regulator with a local FB, when the main loop is disconnected by saturation.
- Any solution which allows this, can be used as an anti-windup.

Digital realization:

- If the controller is implemented digitally, the solution is easy:
- The logic turns off the I term: „if $|u| \geq u_{max}$, $k_I = 0$“

Anti-Windup: Analog solution 1

- (easy to explain, hard to implement – it needs additional non-linearity)



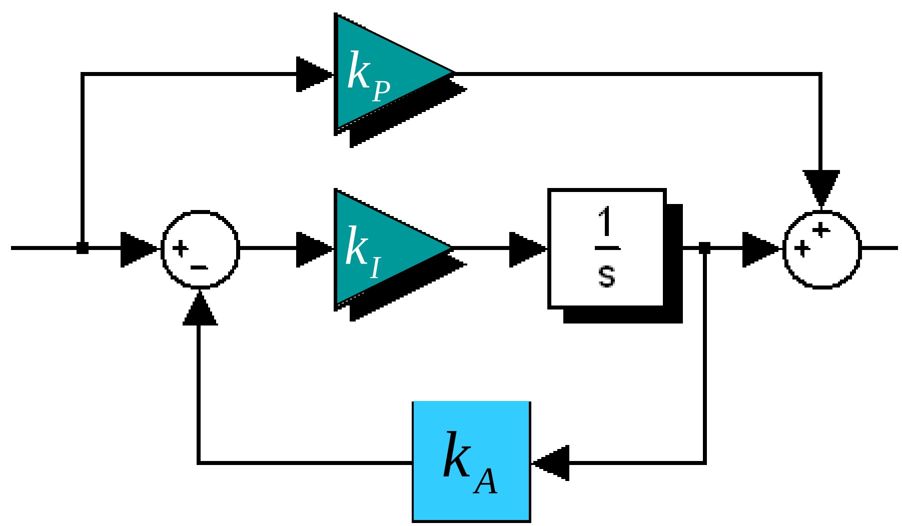
slope k_A

During saturation the circuits are equivalent.

- During saturation the regulator transfer function

$$\frac{k_I}{s + k_I k_A} + k_P$$

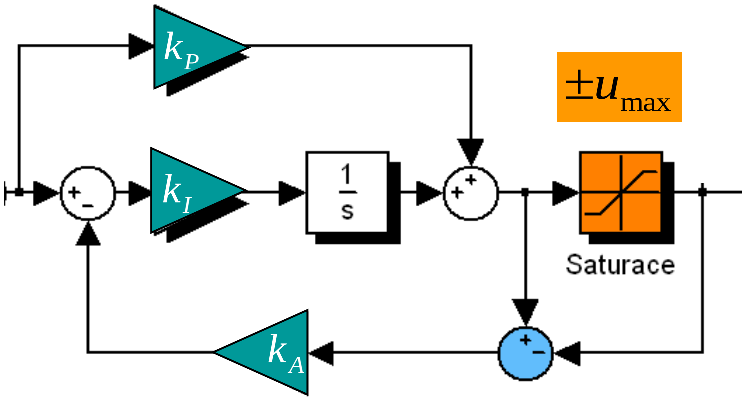
is stabilized.



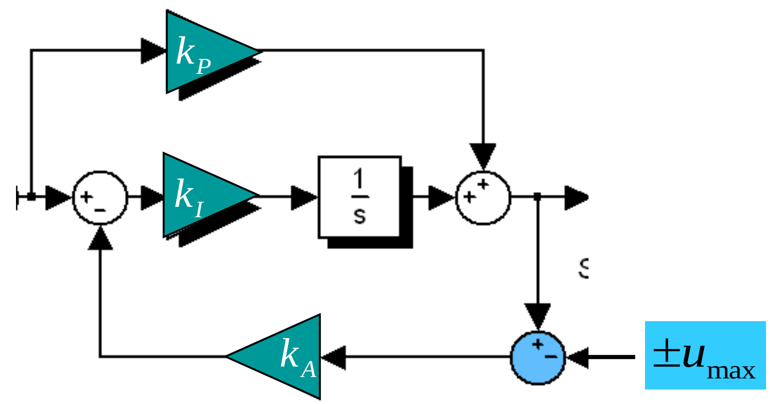
- When saturation ends the added FB opens.
- The regulator is again PI.

Anti-Windup: Analog solution 2

- (hard to explain, easy to implement – it doesn't need additional non-linearity)

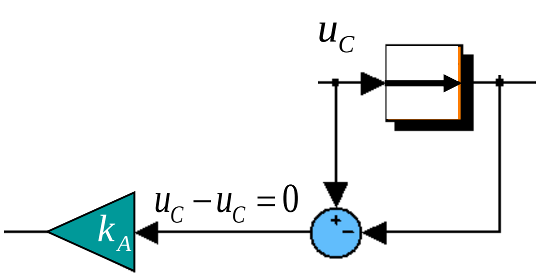


During saturation the circuits are equivalent.



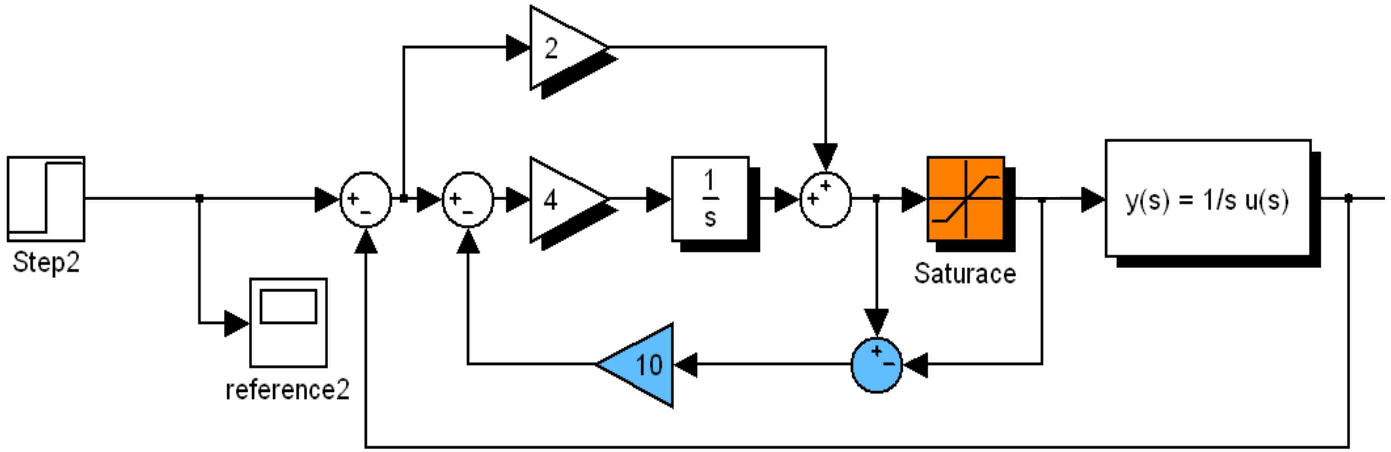
- During saturation the regulator transfer function is

$$\frac{k_P s + k_I}{s + k_I k_A}$$

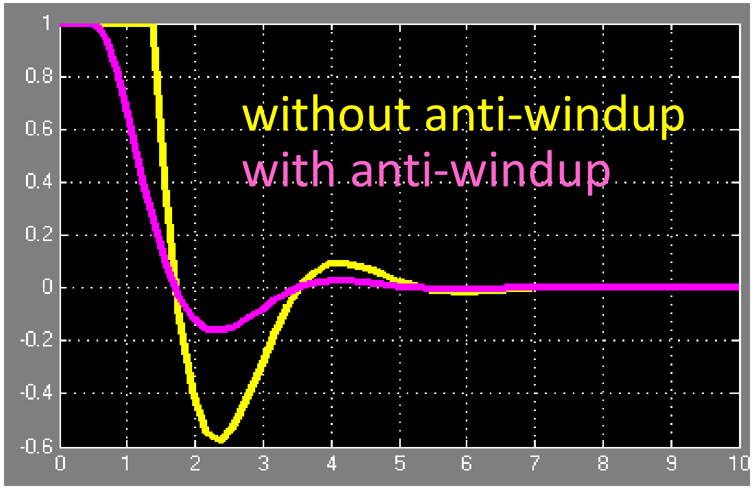


- When saturation ends the added FB opens.
- The regulator is again PI.

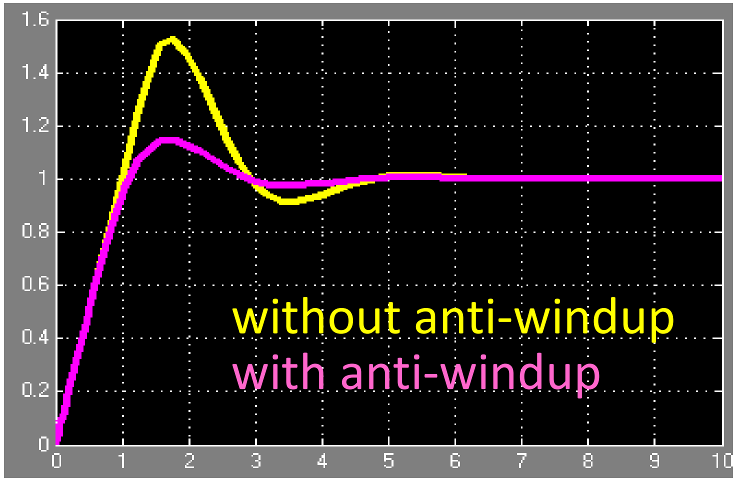
Anti-Windup



2
Step2
4
1
s
Saturace
y(s) = 1/s u(s)
reference2
10



without anti-windup
with anti-windup



without anti-windup
with anti-windup