Frequency based design methods

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Bode's graph based design

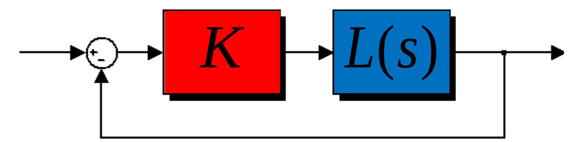
- By shaping frequency response of open loop (OL) – "easy" – indirectly shaping the closed loop (CL) – the aim
- Typically we tune values like ω_C, 0dB crossing, PM, BW, GM

$$L(j\omega) = G(j\omega)D_C(j\omega) \Rightarrow \begin{array}{l} \left|L(j\omega)\right|_{\mathrm{dB}} = \left|G(j\omega)\right|_{\mathrm{dB}} + \left|D_C(j\omega)\right|_{\mathrm{dB}} \\ \arg\left(L(j\omega)\right) = \arg\left(G(j\omega)\right) + \arg\left(D_C(j\omega)\right) \end{array}$$

- Disadvantages: practically we are not able to detect instability, it must be guaranteed from "outside" – these methods can't be used for stabilization of unstable systems – therefore these methods are used mainly/exclusively for stable (usually minimum faze) systems => also CL is stable (according to Nyquist's criterion)
- Not show stopper, representation of Bode's graph for unstable system is questionable (It's not possible to measure, formally it is possible to draw it,…)
- Frequency based method are suited for measured frequency response: We can do the graphical design without mathematical model
- The time delay is not issue for such a design

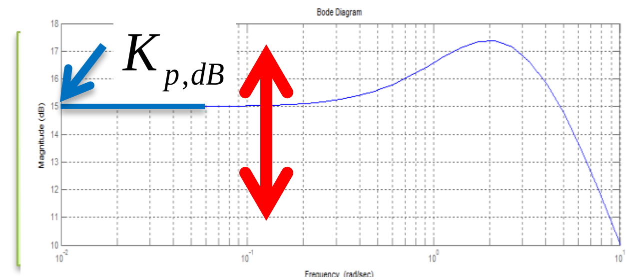
Gain for required study state behavior

Aim: Tune control law gain in such a way, that study state error (for step command) is lower than preselected value. Usually expressed by position constant K_p



$$e_{\text{step,ss}} = 1/(1+K_p)$$

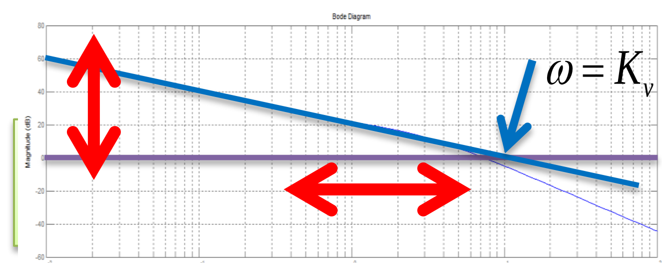
1. Measure/read “initial” study gain
2. The different between required and measured gain is control law gain.



Aim: Tune control law gain in such a way, that study state error (for ramp command) is lower than preselected value. Usually expressed by velocity constant K_v

$$e_{\text{ramp,ss}} = 1/K_v$$

1. Compute required crossing point of low frequency asymptote with „0dB level“
2. Tune the gain to shift the frequency respons towards the crossing point

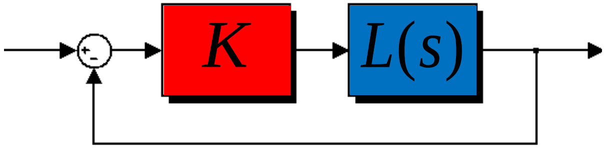


Check CL Stability!

Control law gain tuning for required *PM*

Aim:

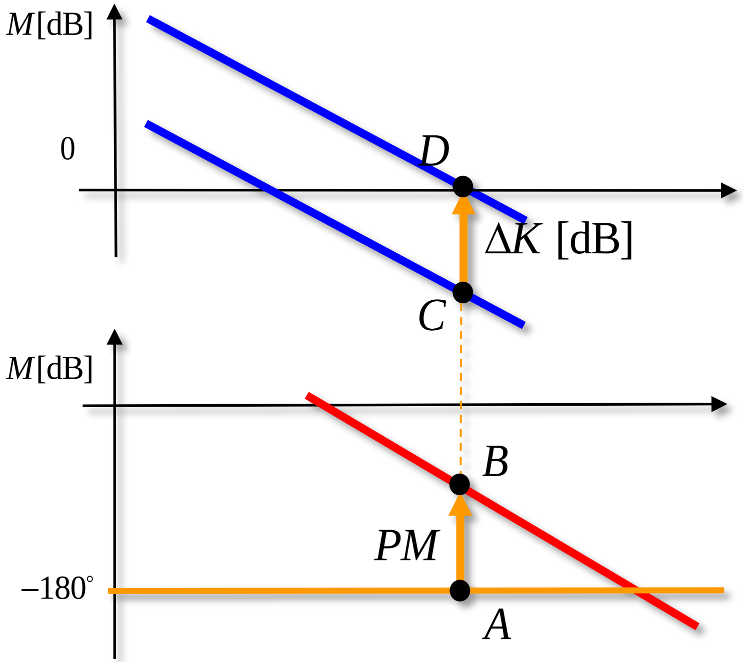
Tune control law in such a way to ensure CL *Phase Margine* (thereby overshoot, damping)



$$PM = \arctan\frac{2\varsigma}{\sqrt{-2\varsigma^2+\sqrt{1+4\varsigma^4}}}, \zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2+\ln^2(\%OS/100)}}$$

Algorithm (for sable L!)

1. Define PM, or compute it form $\%OS$ or ζ
2. Draw Bode's graph for arbitrary selected K
3. Find ω_{PM} frequency on phase graph , where phase is equal to -180° + required PM
4. Change gain in such a way that amplitude for particular frequency is equal to 1 (0 dB), so the $\omega_c = \omega_{PM}$



PI control law tuning

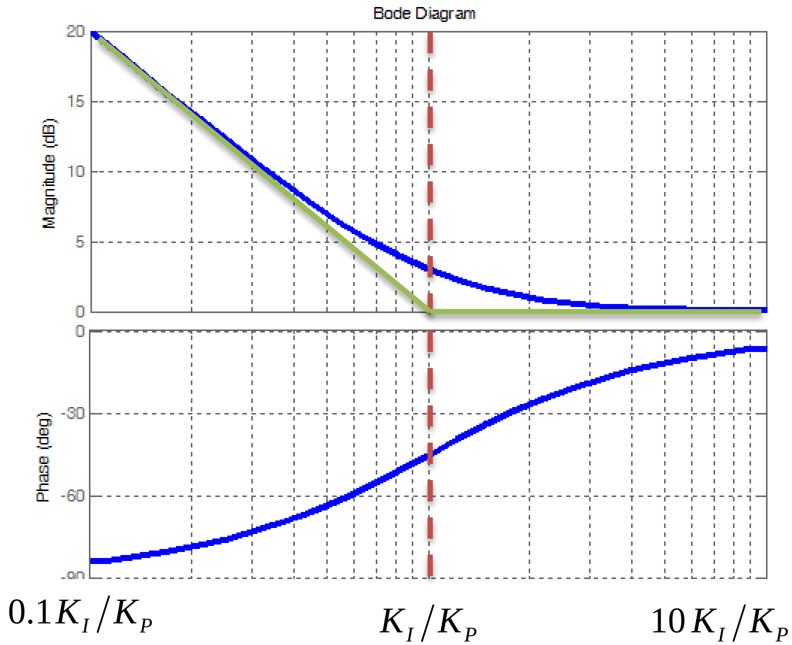
PI – Proportional – Integral controller

$$D_C(s) = K_P + \frac{K_I}{s} = K_P\left(1 + \frac{K_I}{sK_P}\right)$$

- Low pass filter:

$$\left|D_C(j\infty)\right|_{\mathrm{dB}} = 20\log K_P < 0 \quad \text{pro} \quad K_P < 1$$

- Design is done by tuning K_p in such a way that new ω_c ensure better PM
- The cut-off frequency is than determined by gain K_I . Proper choice is at the "left", to avoid phase lag around ω_c .



$$K_I/K_P = \omega_{c,new}/10$$

- PI increase T_r , GM , PM and M_r and decrease ω_{BW} , and OS
- Advantage is high frequency noise attenuation.

PD control law tuning

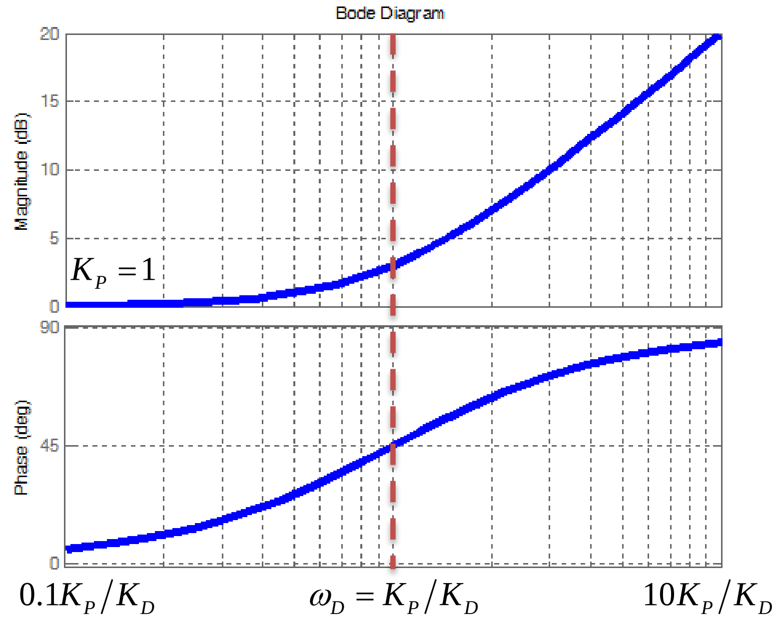
PD – Proportional – derivative controller

$$D_C(s) = K_P + K_D s = K_P\left(1 + \frac{K_D}{K_P}s\right)$$

- High pass filter
- Increase phase on higher frequencies (phase lead), is used for increase of PM,
- also increase gain on higher frequencies, thereby shifting ω_c
- Cut-off frequency is selected by gains:

$$\omega_D = K_P / K_D$$

in such a way that PM is increased.



$$\varphi(\omega_D) = 45^\circ$$

$$|D_C(j\omega_D)| = K_D\omega_D\sqrt{2}$$

- PD controller increase ω_{BW}, GM and M_r ans decrease T_r, and OS.
- Disadvantage is high frequency noise magnification
- Issue: practical realization is slightly different (realization of derivative).

PID control law tuning

PID – Proportional – integral – derivative controller

$$D_C(s) = K_P + \frac{K_I}{s} + K_D s = \left(1 + K_{D1} s\right)\left(K_{P2} + \frac{K_{I2}}{s}\right)$$

$$K_P = K_{P2} + K_{D1} K_{I2}$$

$$K_D = K_{D1} K_{P2}$$

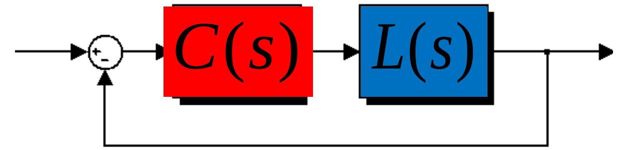
$$K = K_{I2}$$

Design

- First design PD part then PI part
- Or vice versa

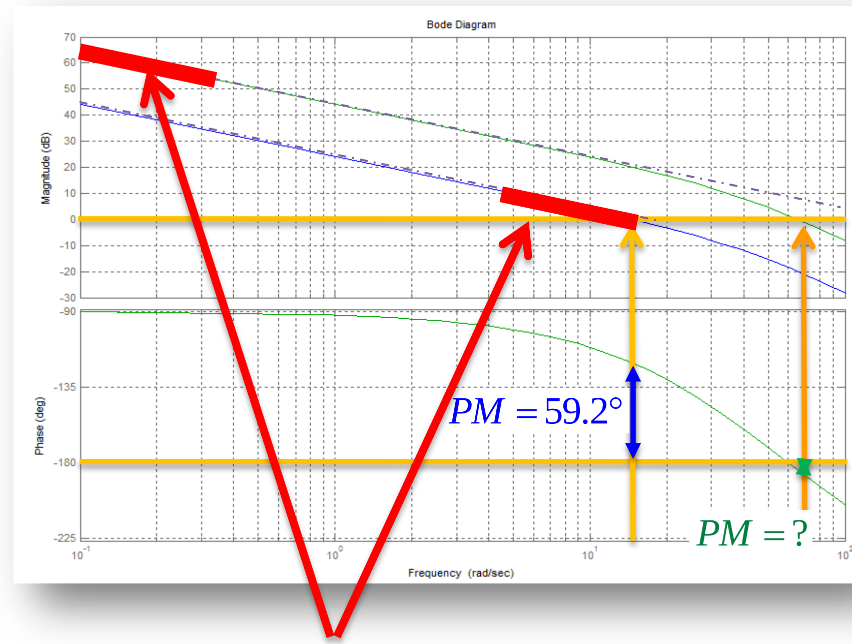
Lag compensation for required PM and K_v

Aim: Ensure required PM and K_v



Discussion:

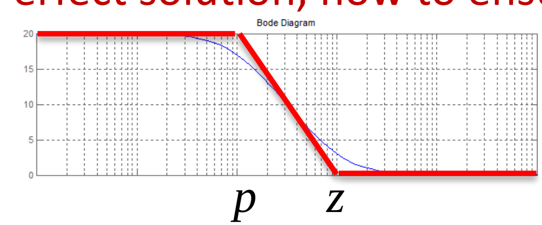
- Required K_v ensure gain according to green graph, but it decrease PM
- On the other hand by decreasing gain (modrý graph), we increase PM, but decrease K_v and thereby we increase ramp tracking error



Conclusion:

- It is not possible to ensure both just by gain
- „lag" type controller is required

Perfect solution, how to ensure it ?



Lag controller

Lag controller (with phase lag)

- Transfer functio $D_C(s)$, or alternatively $C(s)$

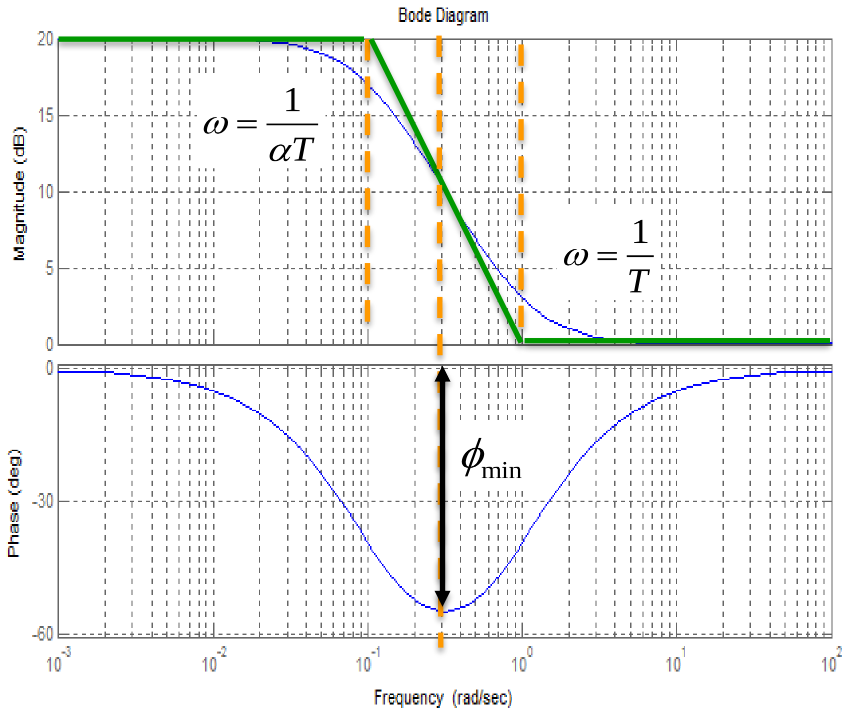
$$D_C(s) = K\frac{s+z}{s+p}, p < z$$

$$C(s) = \alpha\frac{Ts+1}{\alpha Ts+1} = \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}, \quad \alpha > 1$$

where

$$D_C(s) = KC(s), z = 1/T, p = 1/\alpha T$$

- Frequency response



Can be used for:

- Improving study state error by increasing of gain just on low frequencies

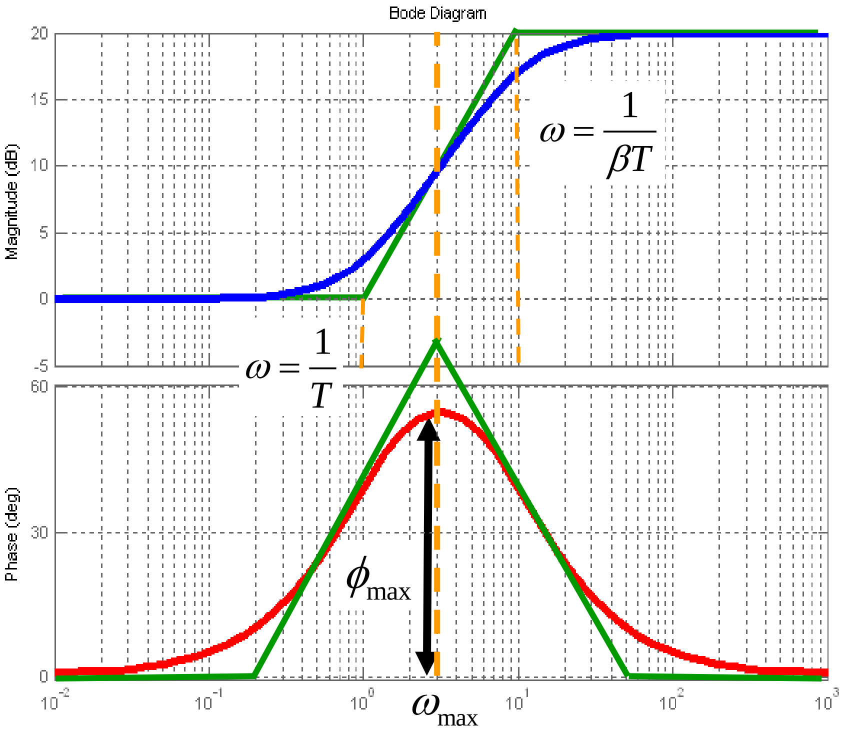
Lead compensator

Lead compensator

$$D(s)=\frac{Ts+1}{\beta Ts+1}=\frac{1}{\beta}\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}},\ \ \beta<1$$

- in this case the DC gain is =1, to preserve already designed study state behavior constant.

- Comparing with PD it has finite gain on high frequencies



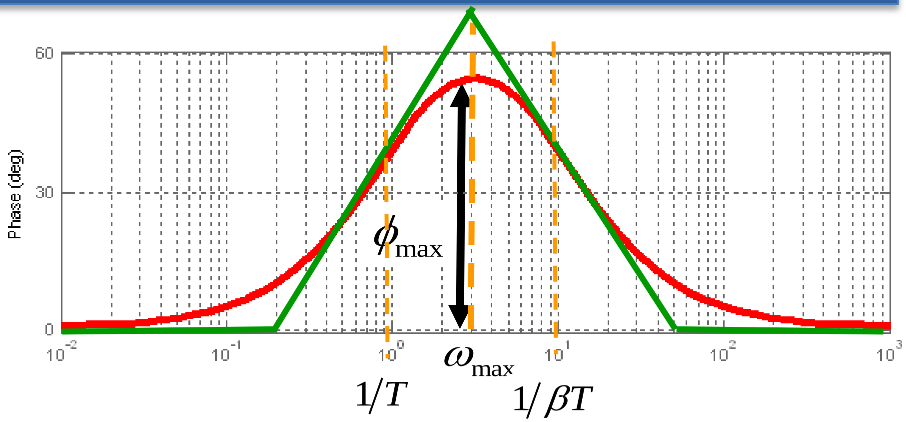
Lead controller

- Let derive equations needed for design
- The phase of controller is

$$\phi = \arctan \omega T - \arctan \omega\beta T$$

- Let derive by ω

$$\frac{d\phi}{d\omega} = \frac{T}{1+(\omega T)^2} - \frac{\beta T}{1+(\omega\beta T)^2} \overset{!}{=} 0$$



- Phase is maximal at frequency $\omega_{max} = \dfrac{1}{T\sqrt{\beta}}$ for which $D(j\omega_{max}) = \dfrac{j\frac{1}{\sqrt{\beta}}+1}{j\sqrt{\beta}+1}$
- The relationship between phase and β is

$$\phi_{max} = \arctan\frac{1-\beta}{2\sqrt{\beta}} = \arcsin\frac{1-\beta}{1+\beta}$$ ans vice versa $$\beta = \frac{1-\sin\phi_{max}}{1+\sin\phi_{max}}$$

$$|D(\omega_{max})| = 1/\sqrt{\beta}$$