16 – State observer. Output feedback. Separation principle.

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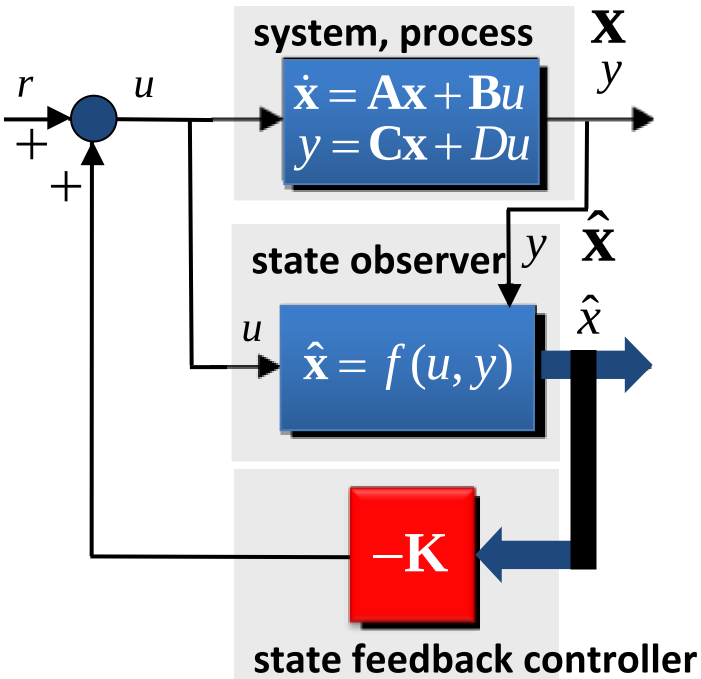
Automatic control 2012

22-III-12

State estimator, observer



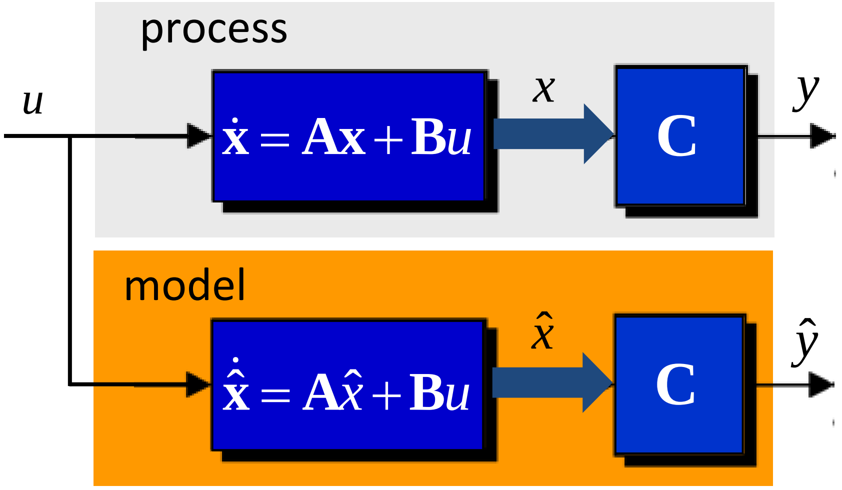
Design goal: $\hat{\mathbf{x}}(t) \approx \mathbf{x}(t), t > 0$ in spite of the fact that the initial condition of the process is unknown and, in addition, unmeasurale disturbances are present and affecting the system (changing its states values...). All inputs and outputs are measured and processed.

Process model as (inadequate) state observer

Attempt I: observer = just the model ...
Obvious drawbacks (estimate's dynamics is just given, disturbances are not covered, ...)

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}u$$

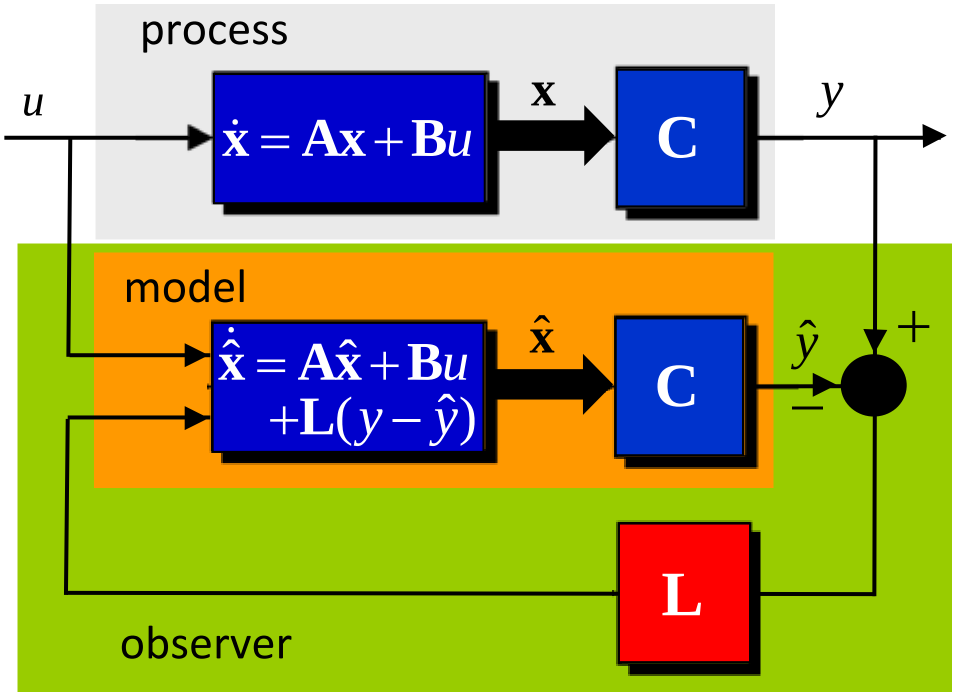


$$\tilde{\mathbf{x}} = \mathbf{x} - \hat{\mathbf{x}}, \quad \tilde{\mathbf{x}}(0) = \mathbf{x}(0) - \hat{\mathbf{x}}(0)$$

$$\dot{\tilde{\mathbf{x}}} = \mathbf{A}\tilde{\mathbf{x}}$$

Full order observer

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}u + \mathbf{L}(y - \hat{y})$$
$$\hat{\mathbf{y}} = \mathbf{C}\hat{\mathbf{x}}$$



$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}u + \mathbf{L}\mathbf{C}(\mathbf{x} - \hat{\mathbf{x}})$$

$$\tilde{x} = x - \hat{x}$$

$$\dot{\tilde{\mathbf{x}}} = (\mathbf{A} - \mathbf{L}\mathbf{C})\tilde{\mathbf{x}} \qquad \dot{\tilde{\mathbf{x}}} = \mathbf{A}_{\mathrm{poz}}\tilde{\mathbf{x}}$$

Duality of estimation and control ...

$$\mathbf{A}_{\text{poz}} = \mathbf{A} - \mathbf{LC}$$

$$\mathbf{A}_{\text{reg}} = \mathbf{A} - \mathbf{BK}$$

$$\mathbf{A}_{\text{poz}}^T = \left(\mathbf{A} - \mathbf{LC}\right)^T = \mathbf{A}^T - \mathbf{C}^T\mathbf{L}^T$$

control state feedback	estimation state observer
$\mathbf{A}$	$\mathbf{A}^T$
$\mathbf{B}$	$\mathbf{C}^T$
$\mathbf{C}$	$\mathbf{B}^T$
$\mathbf{K}$	$\mathbf{L}^T$

... similarly controllability ⇔ oservability, CTRB ⇔ OBSV matrices, controllability ⇔ observability canonical forms, ...

$$\mathbf{A} = \begin{bmatrix} -a_{n-1} & 1 & \cdots & 0 & 0 \\ -a_{n-2} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -a_1 & 0 & \cdots & 0 & 1 \\ -a_0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \end{bmatrix}$$

Observer design ("how to get L")

- L for (A,C) is equal to K for (A transposed, B transposed) due to duality ...
- therefore e.g.

$$\mathbf{T} = O_{new} O_{old}^{-1}$$

$$\mathbf{F} = \begin{bmatrix} -a_{n-1} & 1 & \cdots & 0 & 0 \\ -a_{n-2} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -a_1 & 0 & \cdots & 0 & 1 \\ -a_0 & 0 & \cdots & 0 & 0 \end{bmatrix} - \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_{n-1} \\ l_n \end{bmatrix} \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \end{bmatrix} = \begin{bmatrix} -(a_{n-1}+l_1) & 1 & \cdots & 0 & 0 \\ -(a_{n-2}+l_2) & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -(a_1+l_{n-1}) & 0 & \cdots & 0 & 1 \\ -(a_0+l_n) & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$\det\left(s\mathbf{I} - (\mathbf{A} - \mathbf{LC})\right) = s^n + \left(a_{n-1} + l_1\right)s^{n-1} + \cdots + \left(a_1 + l_{n-1}\right)s + a_0 + l_n$$

$$\det\left(s\mathbf{I} - \mathbf{A}\right) = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$$

$$\boxed{\mathbf{L} = p_{\text{poz}}(\mathbf{A})\mathcal{O}^{-1}\begin{bmatrix}0 & \dots & 0 & 1\end{bmatrix}^T}$$

State feedback & state observer combo ...

$$u = -\mathbf{K}\hat{\mathbf{x}} + r$$

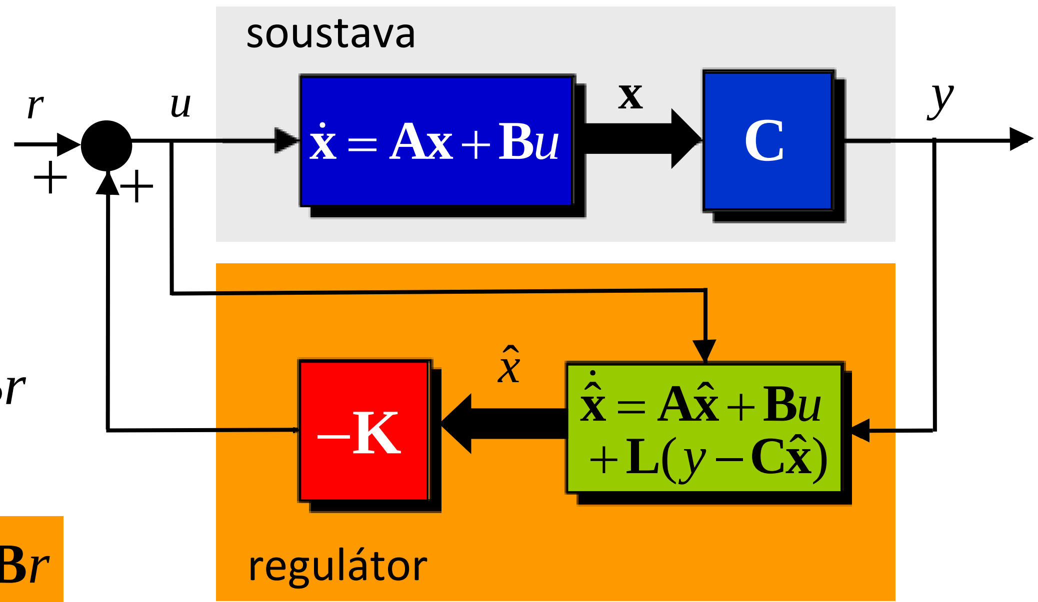
$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u,\ y = \mathbf{C}\mathbf{x}$$

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}u + \mathbf{L}(y - \mathbf{C}\hat{\mathbf{x}})$$

$$u = -\mathbf{K}\hat{\mathbf{x}} + r$$

$$\dot{\hat{\mathbf{x}}} = (\mathbf{A} - \mathbf{L}\mathbf{C})\hat{\mathbf{x}} + \mathbf{B}u + \mathbf{L}y$$

$$= (\mathbf{A} - \mathbf{L}\mathbf{C} - \mathbf{B}\mathbf{K})\hat{\mathbf{x}} + \mathbf{L}y + \mathbf{B}r$$



$$\dot{\hat{\mathbf{x}}} = (\mathbf{A} - \mathbf{L}\mathbf{C} - \mathbf{B}\mathbf{K})\hat{\mathbf{x}} + \mathbf{L}y + \mathbf{B}r$$
$$u = -\mathbf{K}\hat{\mathbf{x}} + r$$

$$u(s) = -\mathbf{K}(s\mathbf{I} - \mathbf{A} + \mathbf{L}\mathbf{C} + \mathbf{B}\mathbf{K})^{-1}\mathbf{L}y(s) + \left(1 - \mathbf{K}(s\mathbf{I} - \mathbf{A} + \mathbf{L}\mathbf{C} + \mathbf{B}\mathbf{K})^{-1}\mathbf{B}\right)r(s)$$

Separation principle

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} - \mathbf{B}\mathbf{K}\hat{\mathbf{x}} + \mathbf{B}r$$
$$\dot{\hat{\mathbf{x}}} = \mathbf{L}\mathbf{C}\mathbf{x} + \left(\mathbf{A} - \mathbf{L}\mathbf{C} - \mathbf{B}\mathbf{K}\right)\hat{\mathbf{x}} + \mathbf{B}r$$

$$\mathbf{x}, \mathbf{e} = \mathbf{x} - \hat{\mathbf{x}}$$

$$\dot{\mathbf{x}} = \left(\mathbf{A} - \mathbf{B}\mathbf{K}\right)\mathbf{x} + \mathbf{B}\mathbf{K}\mathbf{e} + \mathbf{B}r$$
$$\dot{\mathbf{e}} = \left(\mathbf{A} - \mathbf{L}\mathbf{C}\right)\mathbf{e}$$

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{e}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{B}\mathbf{K} & \mathbf{B}\mathbf{K} \\ \mathbf{0} & \mathbf{A} - \mathbf{L}\mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{e} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} r$$

$$u = -\mathbf{K}\hat{\mathbf{x}} + r$$