

# Exercises for lectures

## 16 – Observer and output feedback



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Automatic control 2015



# Example: Pendulum observer – naively

- Pendulum with frequency  $\omega_0$  and equations
- Observer design with double pole

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 0] x$$

$$s_{1,2} = -10\omega_0$$

- Observer characteristic polynomial is

$$p_{\text{poz}}(s) = (s + 10\omega_0)^2 = s^2 + 20\omega_0 s + 100\omega_0^2$$

- Generally

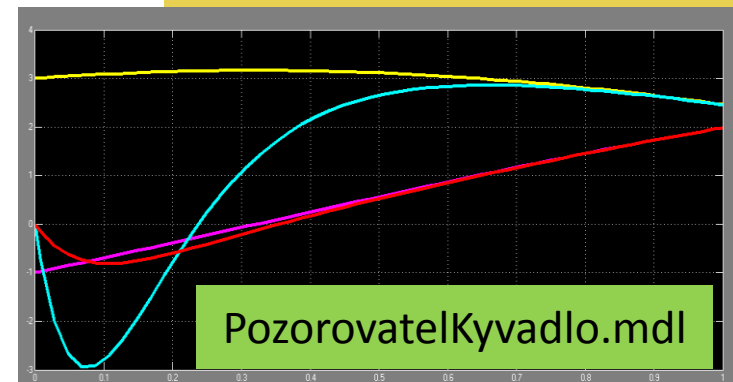
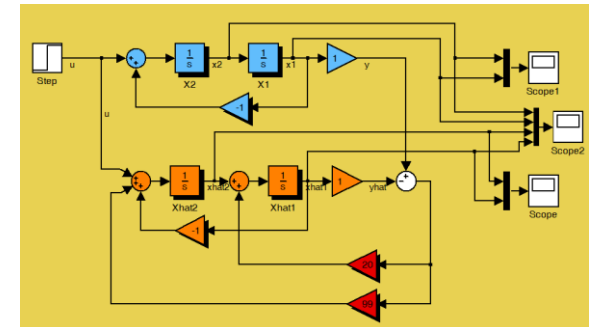
$$\det(s\mathbf{I} - (\mathbf{A} - \mathbf{L}\mathbf{C}))$$

$$= \det \left\{ \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \left( \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix} - \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} [1 \quad 0] \right) \right\}$$

$$= \det \begin{bmatrix} s + l_1 & -1 \\ l_2 + \omega_0^2 & s \end{bmatrix} = s^2 + l_1 s + l_2 + \omega_0^2$$

- From comparison

$$\mathbf{L} = \begin{bmatrix} 20\omega_0 \\ 99\omega_0^2 \end{bmatrix}$$





# Example: Satellite orientation observer

- For system with model

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

- We have already designed state FB placing poles to  $s_{1,2} = 1/\sqrt{2}(-1 \pm j)$
- The transfer function has

$$\omega_n = 1, \zeta = 0.707$$

- We choose  $\omega_n = 5, \zeta = 0.5$  for observer  
Thus

$$s_{3,4} = 5(-0.5 \pm j\sqrt{3}/2) \cong -2.5 \pm j4.3$$

characteristic polynomial

$$p_{\text{poz}} = s^2 + 5s + 25$$

- And calculate

$$L = \begin{bmatrix} 5 \\ 25 \end{bmatrix}$$

```
>> A=[0 1;0 0];B=[0;1];C=[1 0];D=0;
>> G=sdf(A,B,C,D);
>> od=sqrt(3)/2*5;
>> ppoz=(s+2.5+j*od)*(s+2.5-j*od)
ppoz = 25 + 5s + s^2
>> pcpoz=roots(ppoz)
pcpoz =
    -2.5000 + 4.3301i
    -2.5000 - 4.3301i
```

```
>> % DUALITA TAM
>> AA=A';BB=C';CC=B';
>> KK=acker(AA,BB,pcpoz)
KK = 5.0000 25.0000
>> KK=place(AA,BB,pcpoz)
KK = 5.0000 25.0000
```

```
>> % DUALITA ZPATKY
>> L=KK'
L =
    5.0000
   25.0000
>> Apoz=A-L*C
Apoz =
   -5.0000    1.0000
  -25.0000    0
>> eig(Apoz)
ans =
   -2.5000 + 4.3301i
   -2.5000 - 4.3301i
```



# Example: Satellite orientation output FB

- For satellite orientation with equations (Fe5 Ex7.28 s513)

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, y = [1 \quad 0] x \quad G(s) = \frac{1}{s^2}$$

- We have already designed a state FB

$$\mathbf{K} = [1 \quad 1.4142]$$

- And observer (independently)

$$\mathbf{L} = \begin{bmatrix} 5 \\ 25 \end{bmatrix}$$

- Transfer functions for connection are

$$u(s) = \frac{-25 - 40s}{33 + 6.4s + s^2} y(s) + \frac{25 + 5s + s^2}{33 + 6.4s + s^2} r(s)$$

```
>> A=[0 1;0 0];B=[0;1];C=[1 0];D=0
>> G=sdf(A,B,C,D);
>> b=G.num;a=G.den;
>> ro=1;aopt=spf(a*a'+ro*b*b');
>> pc=roots(aopt);
>> K=place(A,B,pc);
>> od=sqrt(3)/2*5;
>> pcpoz=[-2.5+j*od,-2.5-j*od];
>> AA=A';BB=C';CC=B';
>> L=(place(AA,BB,pcpoz)).';
```

```
Areg=A-B*K-L*C;
Fy=sdf(Areg,L,-K)
Fy =
      -25 - 40s
-----
      33 + 6.4s + s^2
>> Fr=1-sdf(Areg,B,K)
Fr =
      25 + 5s + s^2
-----
      33 + 6.4s + s^2
>> roots(Fy.den)
ans =
-3.2071 + 4.7734i
-3.2071 - 4.7734i
```

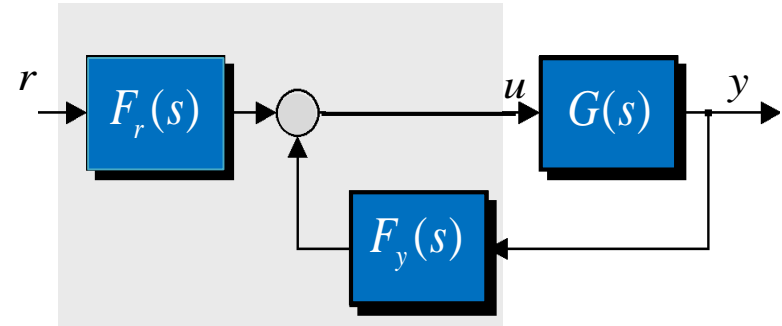


- let's connect the controller with the system.

$$F_y(s) = \frac{-25 - 40s}{33 + 6.4s + s^2}$$

$$G(s) = \frac{1}{s^2}$$

$$F_r(s) = \frac{25 + 5s + s^2}{33 + 6.4s + s^2}$$



- The total transfer function is

$$F_{ry\text{ celk}}(s) = \frac{G(s)F_r(s)}{1 - G(s)F_y(s)}$$

- Where are the observer poles?

```
>> Frycelk=G*Fr/(1-G*Fy)
Frycelk =
      8.3e+2s^2+3.3e+2s^3+90s^4+11s^5+s^6
-----
8.3e+2s^2+1.5e+3s^3+1.4e+3s^4+4.6e+2s^5+1.1e+2s^6+13s^7+s^8
>> Frycelk=coprime(G*Fr/(1-G*Fy))
Frycelk =
      1
-----
      1 + 1.4s + s^2
>> roots(Frycelk.den)
ans =
-0.7071 + 0.7071i
-0.7071 - 0.7071i
```



- We can also calculate the total transfer function from the total system state equations.

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{e}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{BK} & \mathbf{BK} \\ \mathbf{0} & \mathbf{A} - \mathbf{LC} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{e} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} r$$
$$y = \begin{bmatrix} \mathbf{C} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{e} \end{bmatrix}$$

- The result is the same
- Let's try to calculate controllability and observability matrix.
- The total system is observable, but it is not controllable, because
- The observer is not controllable!

```
>> Acelk=[A-B*K,B*K;zeros(2,2),A-L*C]
Acelk =
     0     1.0000         0         0
    -1.0000    -1.4142     1.0000    1.4142
         0         0    -5.0000     1.0000
         0         0   -25.0000         0

>> Bcelk=[B;zeros(2,1)]
Bcelk =
     0
     1
     0
     0

>> Ccelk=[C,zeros(1,2)]
Ccelk =
     1     0     0     0

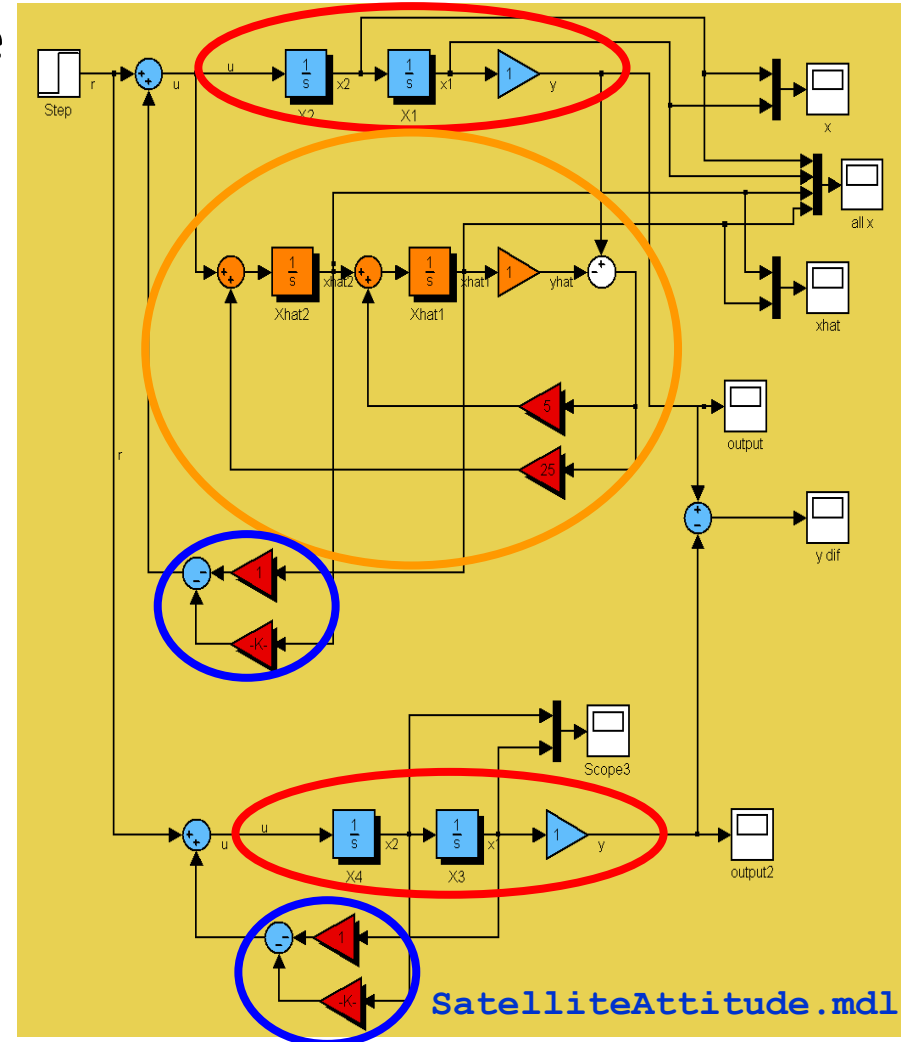
>> Frycelk2=sdf(Acelk,Bcelk,Ccelk)
Frycelk2 =
         1
-----
1 + 1.4s + s^2
```

```
>> obsv(Acelk,Ccelk)
ans =
     1.0000         0         0         0
         0     1.0000         0         0
    -1.0000    -1.4142     1.0000    1.4142
     1.4142     1.0000   -41.7696   -1.0000

>> ctrb(Acelk,Bcelk)
ans =
         0     1.0000    -1.4142     1.0000
     1.0000    -1.4142     1.0000     0.0000
         0         0         0         0
         0         0         0         0
```



- System and observer outputs are same for the same initial conditions.
- The observer is observable, however it is not controllable.





# Example: Position control – antenna azimuth

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- For a system with transfer function

$$\frac{1325}{s(s+1.71)(s+100)}$$

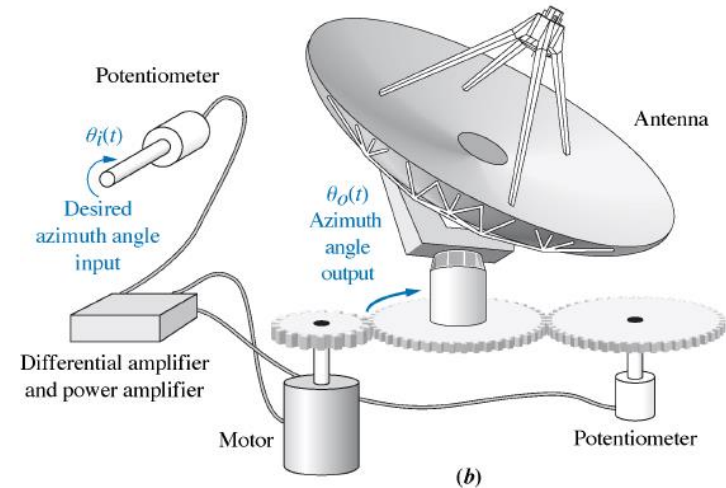
- Design a state controller with specifications

$$OS\% = 10\%, T_s = 1s$$

- Assume that the states can not be measured. Therefore design observer with specifications

$$OS\% = 10\%, \omega_{n,poz} \geq 10\omega_{n,reg}$$

- The optional pole place 10 times more left than the dominant poles are.







## State FB

- From specif.  $OS\% = 10\%$ ,  $T_s = 1\text{s}$  follows  $\zeta = 0.591$ ,  $\omega_{n,reg} = 6.77\text{ rad/s}$
- Consequently, the characteristic polynomial has factor
- $s^2 + 8s + 45.8$  and dominant poles are  $s_{1,2} = -4 \pm j5.46$
- The system is 3rd order, so we have to choose one pole
- We place this pole to non-dominant position  $s_3 = -40$
- Therefore, a characteristic polynomial assigned to state FB is

$$(s^2 + 8s + 45.8)(s + 40) = s^3 + 48s^2 + 365.8s + 1832$$

- And find state realization in controllable form

$$A = \begin{bmatrix} -101.71 & -171 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, C = [0 \quad 0 \quad 1325]$$



- The final state matrix is

$$\mathbf{A} = \begin{bmatrix} -(101.71 + k_1) & -(171 + k_2) & -k_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

- With characteristic polynomial

$$\det(sI - (A - BK)) = s^3 + (101.71 + k_1)s^2 + (171 + k_2)s + k_3$$

- By comparison with the desired polynomial

$$s^3 + 48s^2 + 365.8s + 1832$$

- We get a state FB matrix

$$\mathbf{K} = [k_1 \quad k_2 \quad k_3] = [-53.71 \quad 194.8 \quad 1832]$$



## Observer design

- Transform model to observer normal form

$$\mathbf{A} = \begin{bmatrix} -101.71 & -171 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \mathbf{C} = [0 \quad 0 \quad 1325]$$

➔

$$\mathbf{A}_{nfp} = \begin{bmatrix} -101.71 & 1 & 0 \\ -171 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \mathbf{C}_{nfp} = [1 \quad 0 \quad 0]$$

- With transform matrix

$$\mathbf{T} = \mathbf{O}^{-1} \mathbf{O}_{nfp} = \begin{bmatrix} 0 & 0 & 1325 \\ 0 & 1325 & 0 \\ 1325 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ -102 & 1 & 0 \\ 10174 & -102 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1330 \\ 0 & 1330 & 0 \\ 1330 & 134770 & 22658 \end{bmatrix}$$

- With output injectio, the observer characteristic polynomial is

$$\det(s\mathbf{I} - (\mathbf{A}_{nfp} - \mathbf{L}_{nfp} \mathbf{C}_{nfp})) = s^3 + (101.71 + l_1)s^2 + (171 + l_2)s + l_3$$



# Continuation: Antenna position control

- From observer specifications the factor is

$$\begin{aligned} s^2 + (2 \times 0.591 \times 67.7)s + 67.7^2 &= s^2 + 80s + 4583 \\ &= (s + 40 + j54.62)(s + 40 - j54.62) \end{aligned}$$

- The third observer pole is elected 10 times more to the left  $s_3 = -400$
- The total observer characteristic polynomial is

$$(s^2 + 80s + 4583)(s + 400) = s^3 + 480s^2 + 36580s + 1833000$$

- In comparison with

$$s^3 + (101.71 + l_1)s^2 + (171 + l_2)s + l_3 \quad \text{we get}$$

$$\mathbf{L}_{nfp} = \begin{bmatrix} 378.3 \\ 36412 \\ 1833000 \end{bmatrix}$$

- Finally, we have to transform injection matrix to the original coordinates.

$$\mathbf{L} = \mathbf{T}\mathbf{L}_{nfp} = \begin{bmatrix} 1494 \\ -1.57 \\ 0.286 \end{bmatrix}$$



# Continuation: Antenna position control

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```
>> A=[-101.71 -171 0; 1 0 0; 0 1 0];
>> B=[1;0;0];C=[0 0 1325];
>> creg=s^3+48*s^2+365.8*s+1832,
    r=roots(creg)
creg = 1.8e+003 + 3.7e+002s + 48s^2 + s^3
r =
-40.0000
-4.0000 + 5.4589i
-4.0000 - 5.4589i
>> K=place(A,B,r)
K = 1.0e+003 *
    -0.0537    0.1948    1.8320
>> cpoz=(s^2+80*s+4583)*(s+400),
rpoz=roots(cpoz)
cpoz = 1.8e+006 + 3.7e+004s + 4.8e+002s^2 + s^3
rpoz = 1.0e+002 *
    -4.0000
    -0.4000 + 0.5462i
    -0.4000 - 0.5462i
>> L=(place(A',C',rpoz))'
L = 1.0e+003 *
    1.4932
    -0.0016
    0.0003
```

```
>> Abig=[A-B*K,B*K;zeros(3,3),A-L*C];
>> Bbig=[B;zeros(3,1)];Cbig=[C,zeros(1,3)];
>> T=sdf(Abig,Bbig,Cbig)
T
    1.3e+003
-----
    1.8e+003 + 3.7e+002s + 48s^2 + s^3
>> step(T)
```

