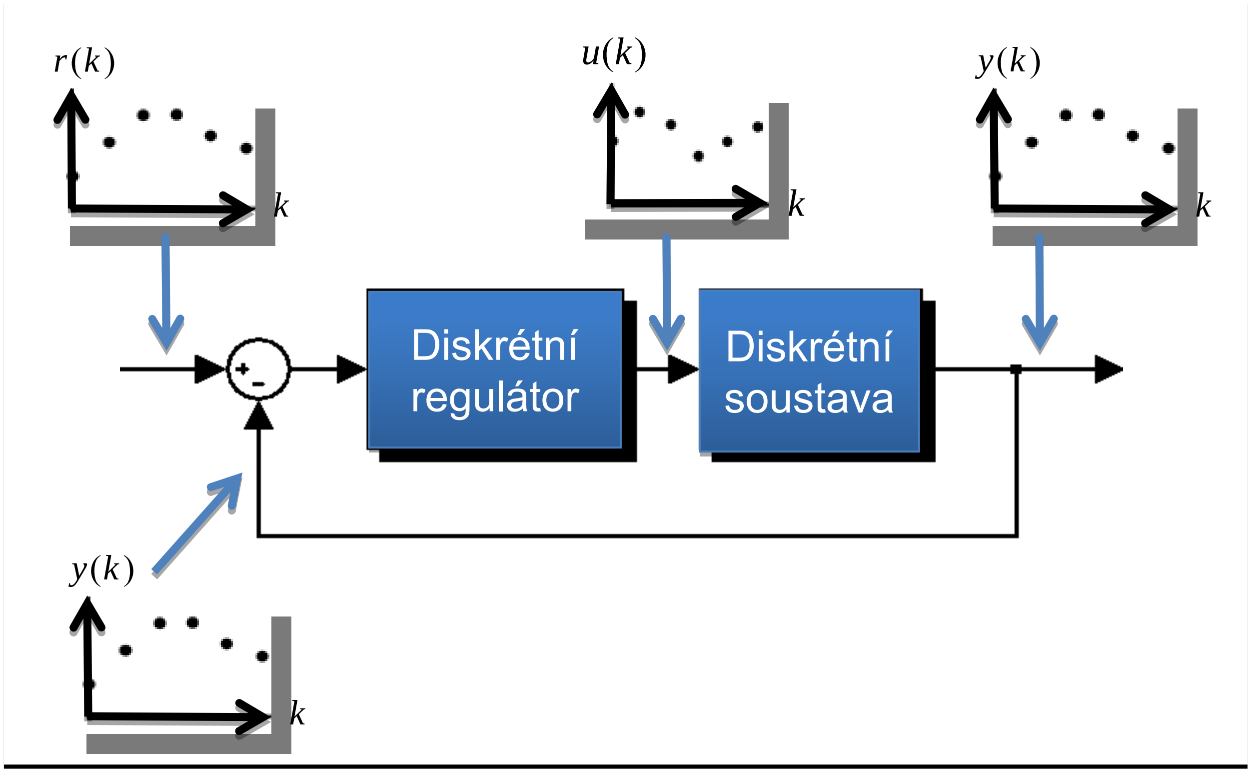
Digital control.
Sampling and reconstruction.
Sampled data systems.

Martin Hromcik.

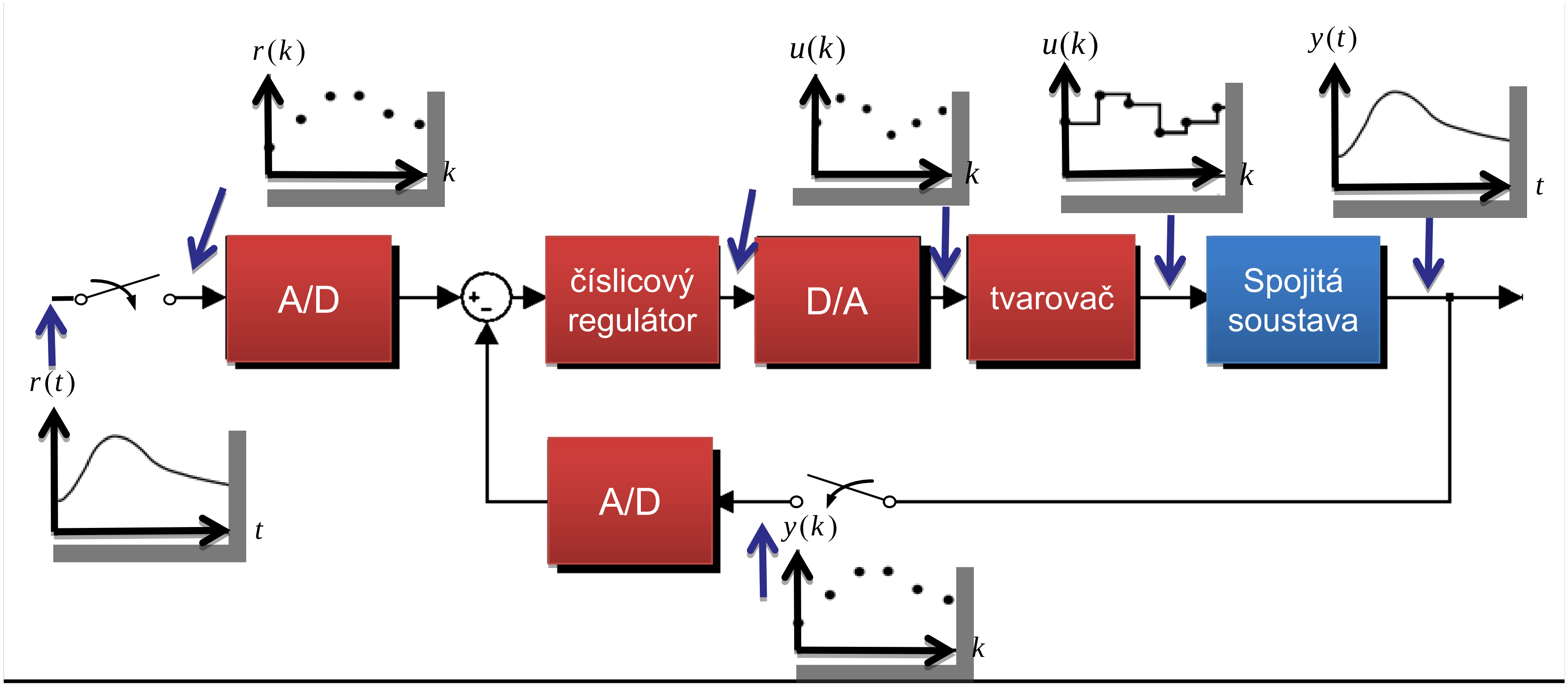
Automatic Control 2012

29-III-12

Discrete-time system & digital controller ...



Digital control & continuous-time systems



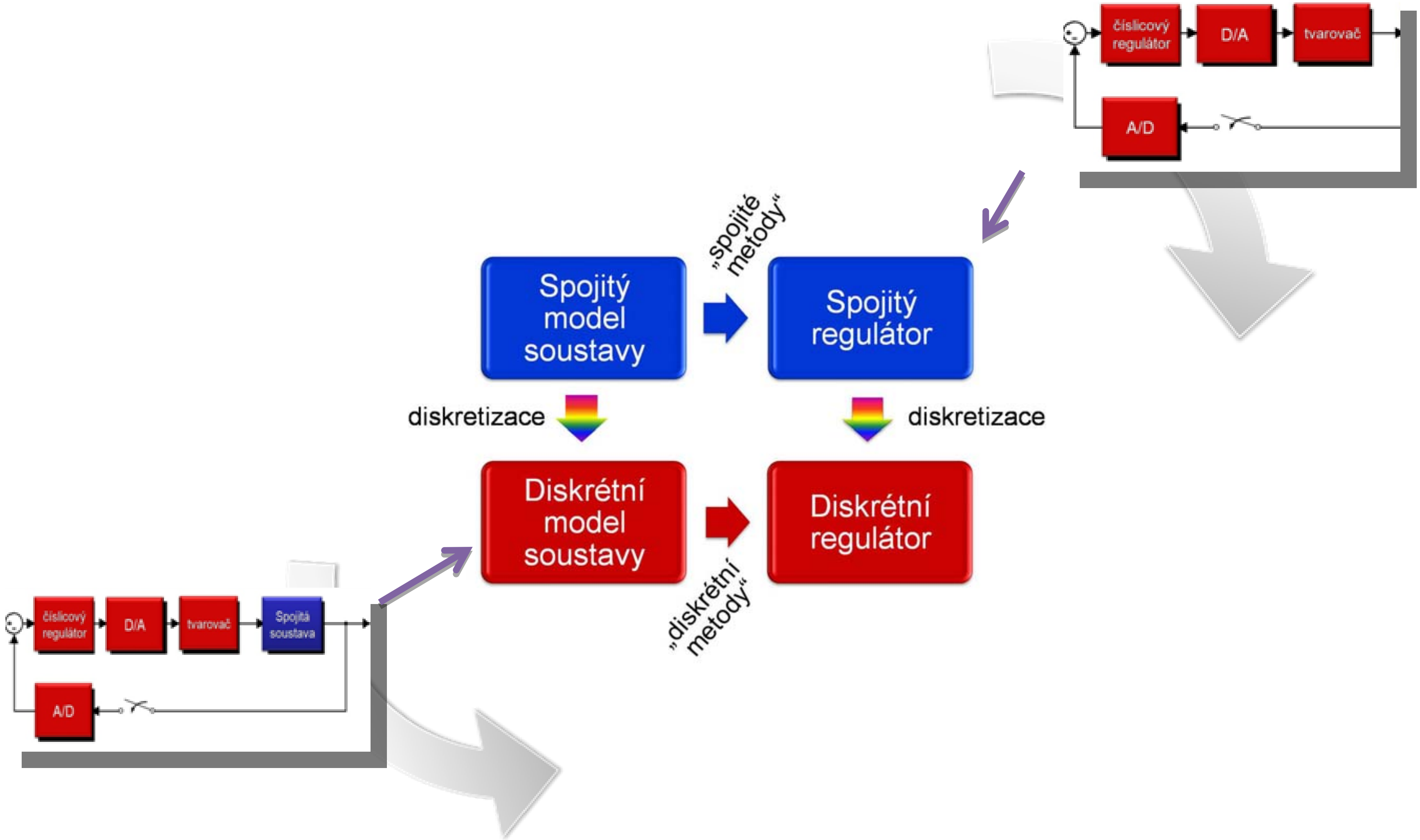
-multi-rate sampling

-hybrid systems (CT plant / DT controls)

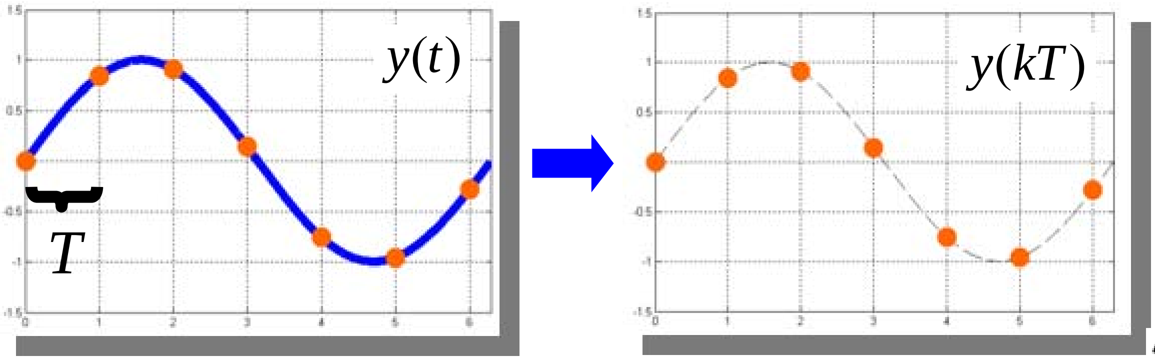
-event-driven sampling (batch-processes)

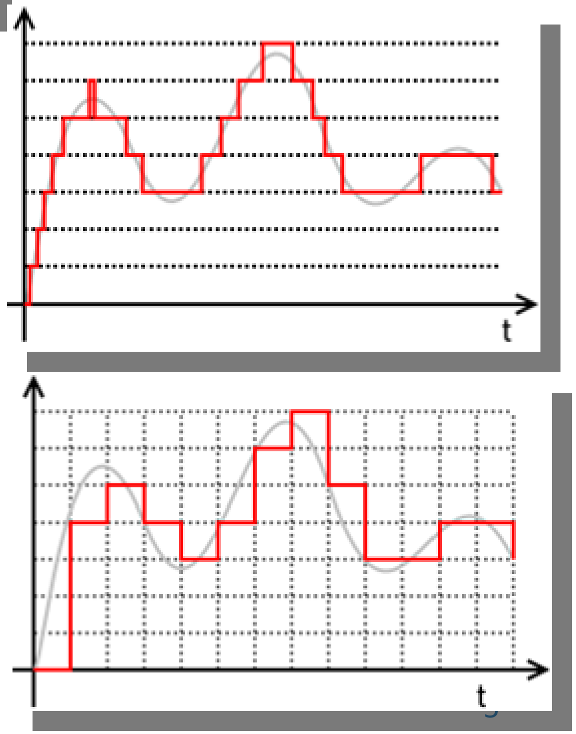
-...

Design options for digital controllers



Sampling

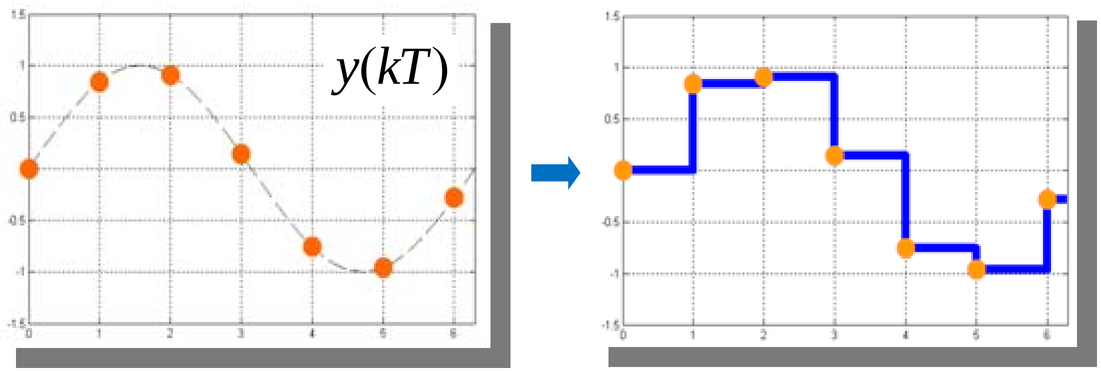


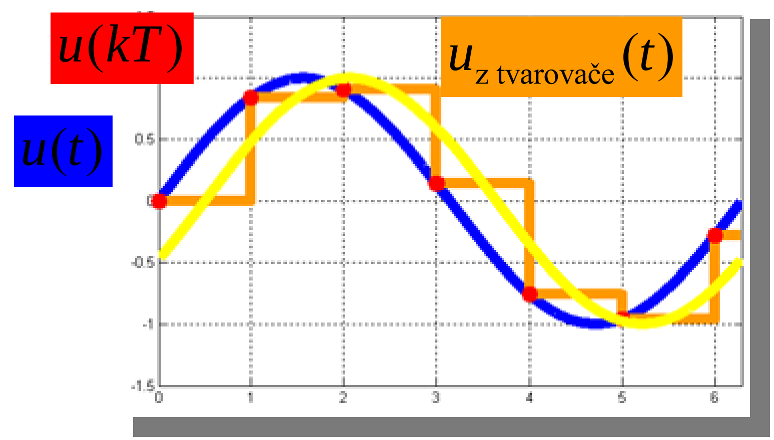


- quantization
- sampling
- digitization

Reconstruction

- reconstrution = holding
- zero-order hold (ZOH)
- ½ Ts delay effect ...





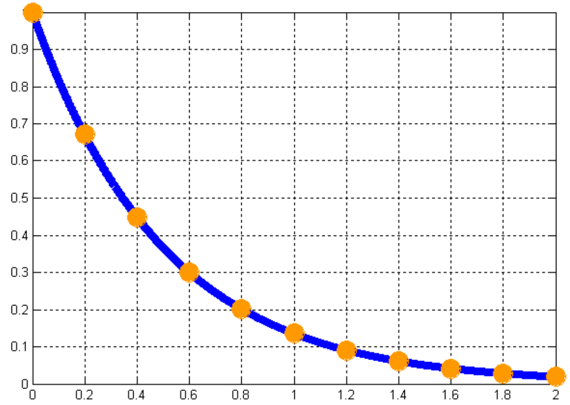
s and *z* variables relationship

$$f(t)=e^{-at}, t>0$$

$$F(s)=\mathcal{L}\{f\}=\frac{1}{s+a}$$

$$f(t)=e^{-akT}$$

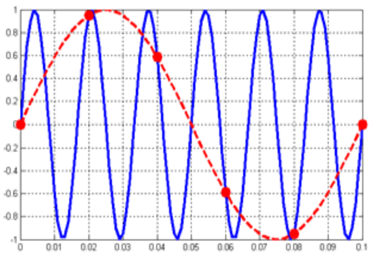
$$F(z)=\mathcal{Z}\{f(kT)\}=\frac{z}{z-e^{-aT}}$$



$$z=e^{sT}$$

Sampling theorem. Aliasing.

- Sampling theorem (Shannon, Nyquist, Kotělnikov,...)
- consequences: CD standard (44.1 kHz), ...
- aliasing (non-control examples: movies with rotating cars' wheels, digital pictures & moire, ...). Solution: anti-aliasing (=low-pass) filter.



$$\omega_s > 2\omega_{\max}$$

$$\omega_N = \omega_S / 2 = \pi / T_s > \omega_{\max}$$

Sampling frequency for control problems

-purely theoretical minimum: $\omega_S > 2\omega_{BW}$

-practical usual solutions: $\omega_S > \left(20 \leftrightarrow 40\right) \times \omega_{BW}$

-alternatively: 5-10 samples per rise-time

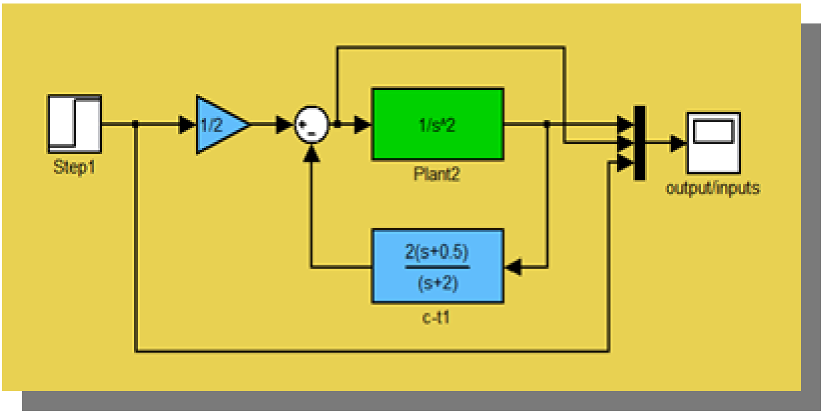
-simulation verification is "a must-do"

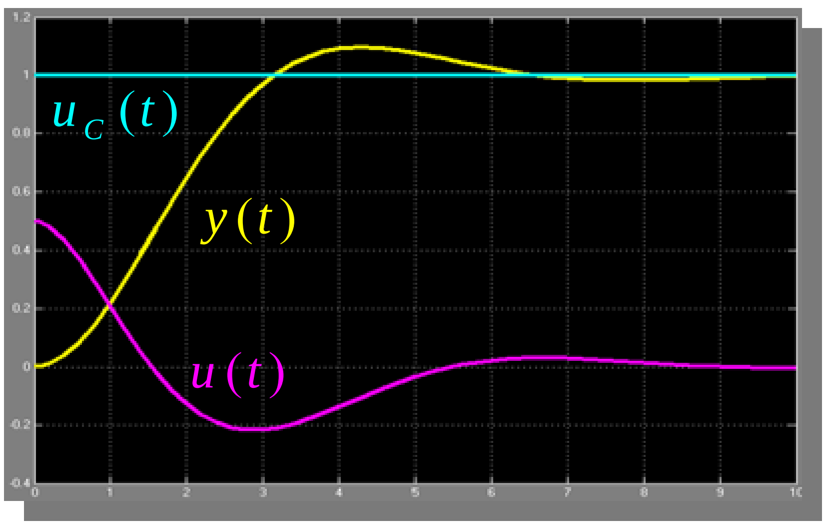
-Ts can range from microseconds (HD drives, audio/active damping applications) to tens-of-minutes/hours (chemical plants, process industry)

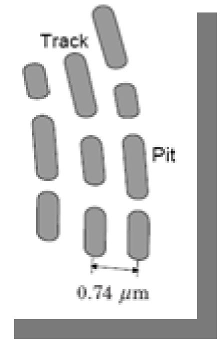
-sampling rates implies potential complexity of the control laws (hence e.g. MPC started in process controls

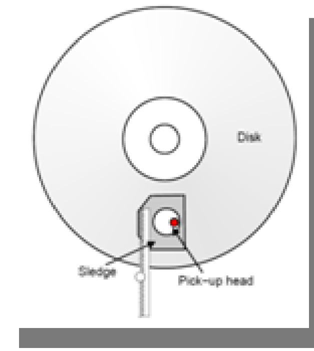
mind: non-causal characteristics of ideal reconstructor. Delay of higher-order holds (1^{st}, 2^{nd} order) unaccaptable for controls typically.

Example (Astrom, Digital Control)

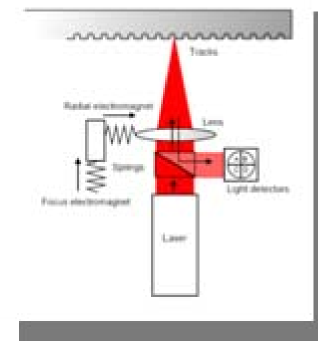








- HD drive arm control / dynamics compensation
- by lead feedback 1st order controller
- the head is modelled as double integrator only



Example (Astrom, Digital Control)

$$u(s) = 0.5u_C(s) - 2\frac{s+0.5}{s+2}y(s) = 0.5u_C(s) - 2y(s) + 2\frac{1.5}{s+2}y(s)$$

$$= 2\left[0.25u_C(s) - y(s) + x(s)\right]$$

$$x(s) = \frac{1.5}{s+2}y(s)$$

$$u(t) = 2\left[0.25u_C(t) - y(t) + x(t)\right]$$

$$\frac{dx}{dt} = -2x(t) + 1.5y(t)$$

$$\frac{x(t+h) - x(t)}{h} = -2x(t) + 1.5y(t)$$

$$u(t_k) = 2\left[0.25u_C(t_k) - y(t_k) + x(t_k)\right]$$

$$x(t_k + h) = x(t_k) + h\left[1.5y(t_k) - 2x(t_k)\right]$$

Example (Astrom, Digital Control)

$$u(t) = 2\left[0.25u_C(t) - y(t) + x(t)\right]$$

$$\frac{dx}{dt} = -2x(t) + 1.5y(t)$$

$$\frac{dx}{dt} \to \frac{x(t+h) - x(t)}{h}$$

$$u(t_k) = 2\left[0.25u_C(t_k) - y(t_k) + x(t_k)\right]$$

$$x(t_k + h) = x(t_k) + h\left[1.5y(t_k) - 2x(t_k)\right]$$

$$u(z) = 2\left[0.25u_C(z) - y(z) + x(z)\right]$$

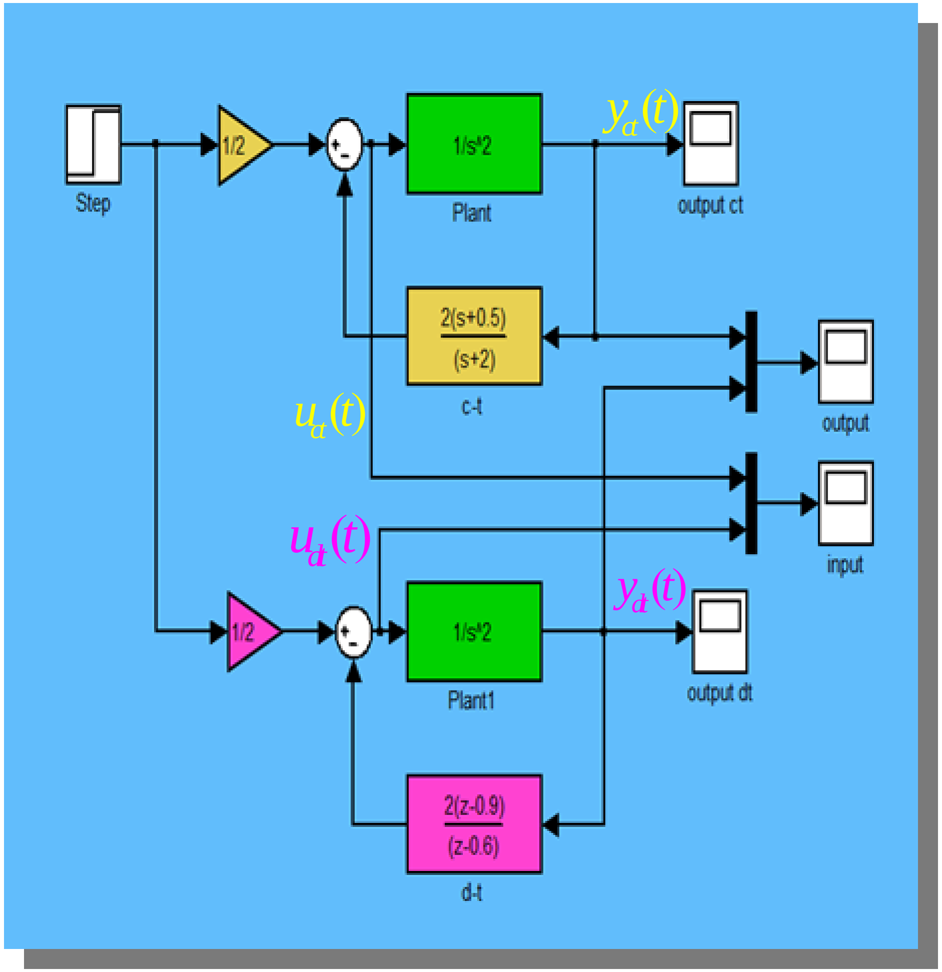
$$zx(z) = x(z) + h\left[1.5y(z) - 2x(z)\right]$$

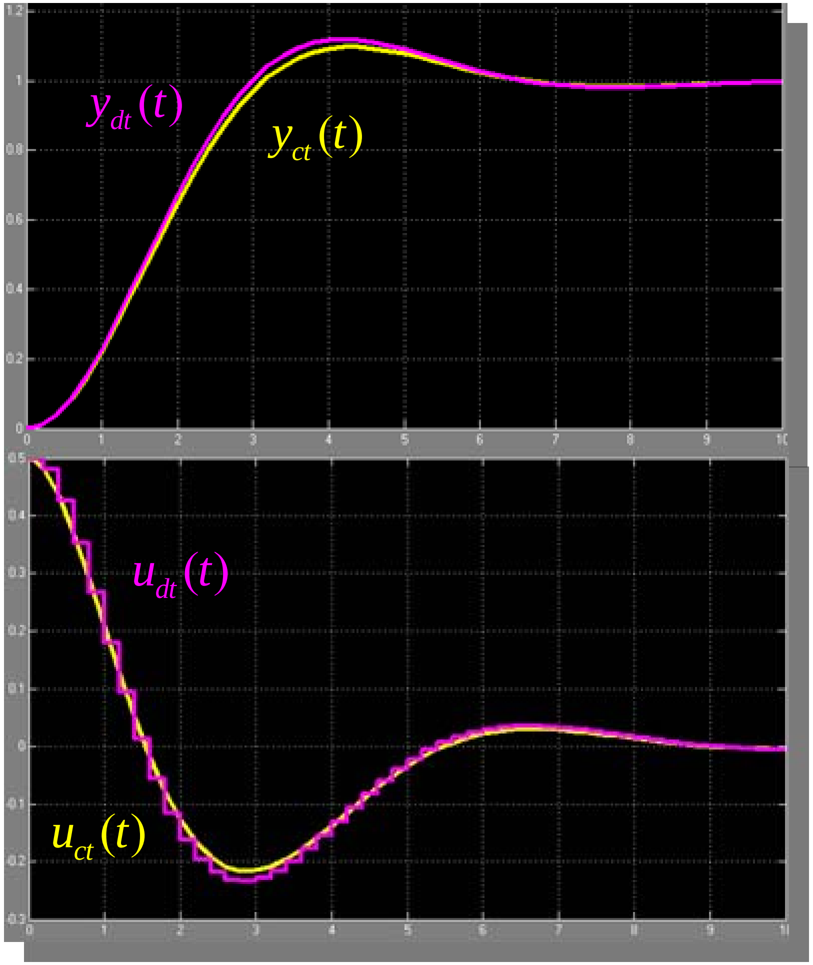
$$u(s) = \frac{1}{2}u_C(s) - 2\frac{s+0.5}{s+2}y(s)$$

$$s \to \frac{z-1}{h}$$

$$u(z) = 0.5u_C(z) - 2\frac{z+0.5h-1}{z+2h-1}y(z)$$

Example (Astrom, Digital Control)





Example (Astrom, Digital Control)

- $h = 0.1, 0.5, 1, 1.5$