Automatic control (AC) course. Basic info.

• location & time
  lectures Mo and We, starting 9:15, room K24
  labs We, starting 11:00, room K26

• web support

• AC team
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• exam
  written + oral + semestral projects, homework, …

• literature
  Franklin, Powell, Emami-Naeini: Feedback Control of Dynamics Systems.

• SW
  Matlab and related …
Control systems: nomenclature


  system / object (BWB A/C ACFA2020)

  model I: FEM. (illustration of the mesh grid)

  model II: set of ODEs. (rigid body flight dynamics)
  \[ \dot{v} + a_{11} v + a_{12} \alpha + a_{13} \theta = c_{11} \delta_T \]
  \[ a_{21} v + \dot{\alpha} + a_{22} \alpha - \dot{\theta} + a_{23} \theta = c_{22} \delta_v \]
  \[ a_{31} v + a_{30} \dot{\alpha} + a_{32} \alpha + \dot{\theta} + a_{33} \theta = c_{32} \delta_v \]

  model III: frequency responses. (rudder to yaw-rate channel)

  - high-complexity: used for control systems performance assessment
  - low-complexity: used for control systems design
Control systems: contributions and goals

- dynamics modification (ACFA 2020 continued ...)

- the rudder channel (open-loop, feedback)
- robust for 12 PAX/FUEL cases
- reduction by >5dB for all cases

mass cases 1:3,4:6

mass cases 1:3,3:4
Control systems: contributions and goals

- reference command tracking (ACFA 2020 continued ...)

- response from roll angle setpoint to roll angle (ROLL AP only, complete FB / FF)
- roll-rate
- response from beta disturbance to beta CG (open-loop, FB)
Control systems: contributions and goals

- measurable disturbance attenuation (ACFA 2020 continued ...)

- alpha (angle-of-attack) signal measured by a probe at the nose
- wind-gusts detected before approaching the wing
- feed-forward controller, with ailerons/flaps as actuators (acting before the disturbance appears)
- two approaches: alpha FIR filtered (Wildschek, Maier), alpha used as trigger (Wildschek, Hanis)
- similar application: active echo cancellation (headphones, car audio, ...)

![Graphs showing measurable disturbance attenuation](image)
Control systems: most common architectures

- **systems of interest**
  - plant, process ... controlled systems
  - controller, compensator, control law
  - closed-loop system, control loop

- **involved signals**
  - input signal: control input / reference signal
  - disturbance: measured / unmeasurable
  - output: controlled / measured
  - internal variables (states)
  - measurement noise
  - control error
Control systems: most common architectures

plant

FB controller

FF controller
Control systems architectures: ACFA 2020 examples

- reference command tracking

- two-stage design (note advantages due to safety, flight-testing, ...):
  - robust roll-AP designed (robust $H_2$ optimal MIMO controller, low order – 6)
  - $H_\infty$ mixed-sensitivity robust MIMO vibrations damper built upon that
- total order – ~25
- robust for 12 mass cases
Control systems architectures: ACFA 2020 examples

- reference command tracking

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- roll-rate
Control systems architectures: ACFA 2020 examples

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Control systems architectures: ACFA 2020 examples
Control systems further nomenclature

- Continuous/discrete time systems, sampling

- SISO - MIMO

- Lumped / distributed parameters (ODE / PDE)
- Time varying / time invariant
- Linear / nonlinear
State-space models

- nonlinear

\[
\dot{x}(t) = f(x(t), u(t), t) \\
y(t) = h(x(t), u(t), t)
\]

- linear time varying (LTV)

\[
\dot{x}(t) = A(t)x(t) + B(t)u(t) \\
y(t) = C(t)x(t) + D(t)u(t) \\
x(t_0^-) = x_0
\]

- linear time invariant (LTI)

\[
\dot{x}(t) = Ax(t) + Bu(t) \\
y(t) = Cx(t) + Du(t) \\
x(0^-) = x_0
\]

- time invariant system
- autonomous system
- static nonlinearity
- equilibrium
- limit cycles
- homogeneity / additivity conditions
- time / frequency domains characteristics
I/O models

- nonlinear

\[ D(y^{(n)}(t), \ldots, \dot{y}(t), y(t), t) = N(u^{(m)}(t), \ldots, \dot{u}(t), u(t), t) \]

- linear time varying (LTV)

\[ a_n(t)y^{(n)}(t) + \cdots + a_1(t)\dot{y} + a_0(t)y(t) = b_m(t)u^{(m)}(t) + \cdots + b_1(t)\dot{u}(t) + b_0(t)u(t) \]

- linear time invariant (LTI)

\[ a_ny^{(n)}(t) + \cdots + a_1\dot{y}(t) + a_0y(t) = b_mu^{(m)}(t) + \cdots + b_1\dot{u}(t) + b_0u(t) \]

- further / related LTI descriptions: transfer function, Bode plots (frequency characteristics), step / impulse responses
- state-space ↔ I/O descriptions, realizations, canonical forms
• local linearization only discussed here
• not exact feedback linearization
• nonlinear state-space model

\[
\begin{align*}
\dot{x}(t) &= f(x(t), u(t), t) \\
y(t) &= h(x(t), u(t), t)
\end{align*}
\]

• around equilibrium (or, more generally, reference trajectory)

\[
\begin{align*}
\dot{x}(t) &= \dot{x}_p(t) + \Delta \dot{x}(t) = f(x_p(t) + \Delta x(t), u_p(t) + \Delta u(t)) \\
&= f(x_p(t), u_p(t)) + \frac{\partial f}{\partial x} \bigg|_{(x_p, u_p)} \Delta x(t) + \frac{\partial f}{\partial u} \bigg|_{(x_p, u_p)} \Delta u(t) \\
y(t) &= y_p(t) + \Delta y(t) = g(x_p(t) + \Delta x(t), u_p(t) + \Delta u(t)) \\
&= h(x_p(t), u_p(t)) + \frac{\partial h}{\partial x} \bigg|_{(x_p, u_p)} \Delta x(t) + \frac{\partial h}{\partial u} \bigg|_{(x_p, u_p)} \Delta u(t)
\end{align*}
\]

-higher-order terms omitted
Linearization

\[ \dot{x}(t) = f(x(t), u(t)) \quad x_p(t), u_p(t) \quad \Delta \dot{x}(t) = A \Delta x(t) + B \Delta u(t) \]
\[ y(t) = h(x(t), u(t)) \quad \Delta y(t) = C \Delta x(t) + D \Delta u(t) \]

\[ A = \left. \frac{\partial f}{\partial x} \right|_{(x_p, u_p)}, B = \left. \frac{\partial f}{\partial u} \right|_{(x_p, u_p)}, C = \left. \frac{\partial h}{\partial x} \right|_{(x_p, u_p)}, D = \left. \frac{\partial h}{\partial u} \right|_{(x_p, u_p)} \]

\[ A = \left. \frac{\partial f}{\partial x} \right|_{(x_p, u_p)} = \begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots \\
\frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots \\
\vdots & \vdots & \ddots \\
\end{bmatrix}_{x=x_p, u=u_p} \]
Linearization

• nonlinear I/O model (higher-order ODE)

\[ D(y^{(n)}(t), \ldots, \dot{y}(t), y(t), t) = N(u^{(m)}(t), \ldots, \dot{u}(t), u(t), t) \]

• around equilibrium (or, more generally, reference trajectory)

\[ y(t) = y_p(t) + \Delta y(t), \ldots, y^{(n)}(t) = y_p^{(n)}(t) + \Delta y^{(n)}(t), \]
\[ u(t) = u_p(t) + \Delta u(t), \ldots, u^{(m)}(t) = u_p^{(m)}(t) + \Delta u^{(m)}(t) \]

\[
\begin{aligned}
D_p + \frac{\partial D}{\partial y} \Delta y + \frac{\partial D}{\partial \dot{y}} \Delta \dot{y} + \cdots + \frac{\partial D}{\partial y^{(n)}} \Delta y^{(n)} &+ \cdots = N_p + \frac{\partial N}{\partial u} \Delta u + \frac{\partial N}{\partial \dot{u}} \Delta \dot{u} + \cdots + \frac{\partial N}{\partial u^{(m)}} \Delta u^{(m)} + \cdots \\
\frac{\partial D}{\partial y} \Delta y + \frac{\partial D}{\partial \dot{y}} \Delta \dot{y} + \cdots + \frac{\partial D}{\partial y^{(n)}} \Delta y^{(n)} &\cong \frac{\partial N}{\partial u} \Delta u + \frac{\partial N}{\partial \dot{u}} \Delta \dot{u} + \cdots + \frac{\partial N}{\partial u^{(m)}} \Delta u^{(m)} \\
\end{aligned}
\]

\[ a_0 \Delta y + a_1 \Delta \dot{y} + \cdots + a_n \Delta y^{(n)} = b_0 \Delta u + b_1 \Delta \dot{u} + \cdots + b_m \Delta u^{(m)} \]
Discrete-time models

- nonlinear SS models
  \[ x(k + 1) = f(x(k), u(k), k) \]
  \[ y(k) = h(x(k), u(k), k) \]

- linear SS models
  \[ x(k + 1) = A(k)x(k) + B(k)u(k) \]
  \[ y(k) = C(k)x(k) + D(k)u(k) \]

- initial conditions
  \[ x(k_0) = x_0 \]
  \[ x(0) = x_0 \]

- equilibrium
  \[ u_e(k) = u_e, \ x_e(k) = x_e \implies x_e = f(x_e, u_e) \]
Discrete-time models

• nonlinear IO model

\[ D(y(k+n), \ldots, y(k+1), y(k), k) = N(u(k+m), \ldots, u(k+1), u(k), k) \]

• linear IO model

\[
\begin{align*}
  a_n(k)y(k+n) + \cdots + a_1(k)y(k+1) + a_0(k)y(k) \\
  = b_m(k)u(k+m) + \cdots + b_1(k)u(k+1) + b_0(k)u(k)
\end{align*}
\]

• LTI IO model (relate to transfer function, Z-transform)

\[
\begin{align*}
  a_n y(k+n) + \cdots + a_1 y(k+1) + a_0 y(k) \\
  = b_m u(k+m) + \cdots + b_1 u(k+1) + b_0 u(k)
\end{align*}
\]

• initial conditions, equilibrium

• linearization

\[
\begin{align*}
  \Delta x(k+1) & \approx \frac{\partial f}{\partial x}(x_p, u_p) \Delta x(k) + \frac{\partial f}{\partial u}(x_p, u_p) \Delta u(k) \\
  \Delta y(k) & \approx \frac{\partial h}{\partial x}(x_p, u_p) \Delta x(k) + \frac{\partial h}{\partial u}(x_p, u_p) \Delta u(k)
\end{align*}
\]