11 – Industrial controllers

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Automatic control
Impulse controllers

\[ u = \begin{cases} 
  u_{\text{max}}, & \text{if } e > 0 \\
  u_{\text{min}}, & \text{if } e < 0 
\end{cases} \]
### PID controllers

The PID controller is defined as:

\[
u(t) = k_p e(t) + k_i \int_{t_0}^{t} e(\tau)d\tau + k_d \dot{e}(t)
\]

The transfer function of the PID controller is:

\[
\frac{U(s)}{E(s)} = D_C(s) = k_p + \frac{k_i}{s} + k_d s
\]

The transfer function of the PID controller is also given by:

\[
D_C(s) = k_p \left[ 1 + \frac{1}{T_i s} + T_D s \right]
\]

Where:
- \( k_p \) is the proportional gain
- \( k_i \) is the integral gain
- \( k_d \) is the derivative gain
- \( T_i \) is the integral time
- \( T_D \) is the derivative time

**Diagram:**
- The diagram illustrates the control action of the PID controller over time, showing the influence of past, present, and future errors on the control output.
- The graph depicts the response of the system to a step input, highlighting the transient and steady-state behavior of the system.

**References:**
- Michael Šebek
- ARI-11-2012

**Note:**
- The mathematical expressions and diagrams are intended to illustrate the concepts of automatic control and their implementation in practical systems.
"Industrial" PID’s

\[ u(s) = k_p \left[ 1 + \frac{1}{T_i s} + T_D s \right] e(s) \]

- additional parameters on top of the basic formula (defining various limiters, high freq. roll off, dead-zones, saturations, anti-windup measures etc.). For example:

\[ u(s) = k \left[ (b y_{sp}(s) - y(s)) + \frac{1}{T_i s} (y_{sp}(s) - y(s)) - \frac{T_D s}{1 + T_D s / N} y(s) \right] \]

- filtered derivative part (limited gain for high freqs)
- D attached to output only

≈ \( N \in [3, 20] \)

- setpoint weighting (reference and output separation; \( b=0...1 \))

- Alternative ("industry-common") formulation:

\[ u(s) = k'_p \left( 1 + \frac{1}{T'_i s} \right) \left( 1 + T'_D s \right) e(s) \]
Ziegler-Nichols experimental PID tuning

1st method …

2nd method …

\[ k_U, P_U \rightarrow \]

\[ P \]
\[ k_p = 1/A \]
\[ k_p = 0.9/A, T_i = 3L \]
\[ k_p = 1.2/A, T_i = 2L, T_D = 0.5L \]

\[ PI \]
\[ P_i = 0.5k_U \]
\[ k_p = 0.45k_U, T_i = P_U/1.2 \]
\[ k_p = 0.6k_U, T_i = P_U/2, T_D = P_U/8 \]

\[ PID \]
\[ k_p = 0.5k_U \]
Lead controller (i.e. phase-lead)
~ PD
\[ p > z \quad \text{and} \quad -p < -z \]

Lag controller (i.e. phase lag)
~ PI
\[ p < z \quad \text{and} \quad -p > -z \]