5 – Systems identification trivia.

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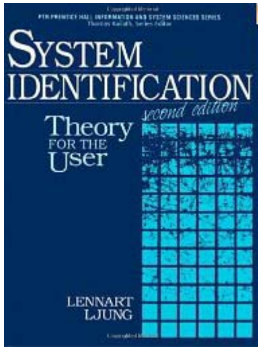
Systems identification

- white box
- grey box
- black box

Open-loop
Time domain
Off-line

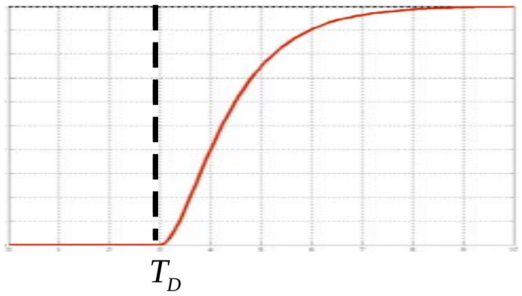
Advanced follow-ups (covered in further courses)

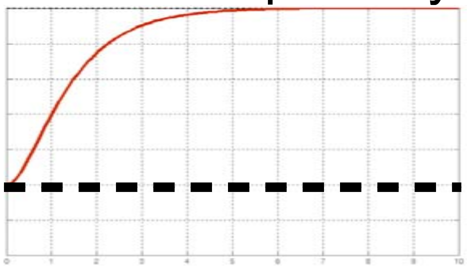
- Recursive
- On-line
- Stochastic (ARX, ARMAX, subspace methods, …)
- Closed-loop
- L. Ljung: System Identification: Theory for the User (2nd Ed.) Prentice Hall, 1999. ISBN 978-0136566953
- Matlab: System Identification Toolbox



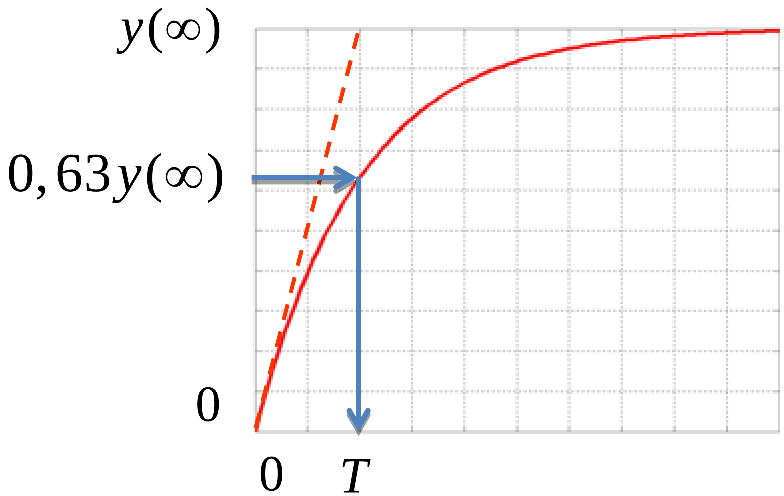
Bump test

1. Experimentally measured step response
2. Interpolation. Parameters fitting. “Visual” fit.
3. Time-delays and offsets treated separately





$$G(s) = \frac{k}{1+Ts}$$

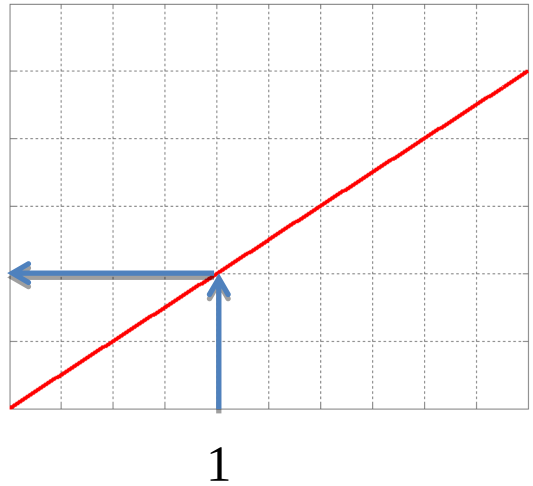


$$k = \frac{y(\infty)}{u(\infty)}$$

Bump test

$$G(s) = \frac{1}{T_i s}$$

$$y = \frac{1}{T_i}$$

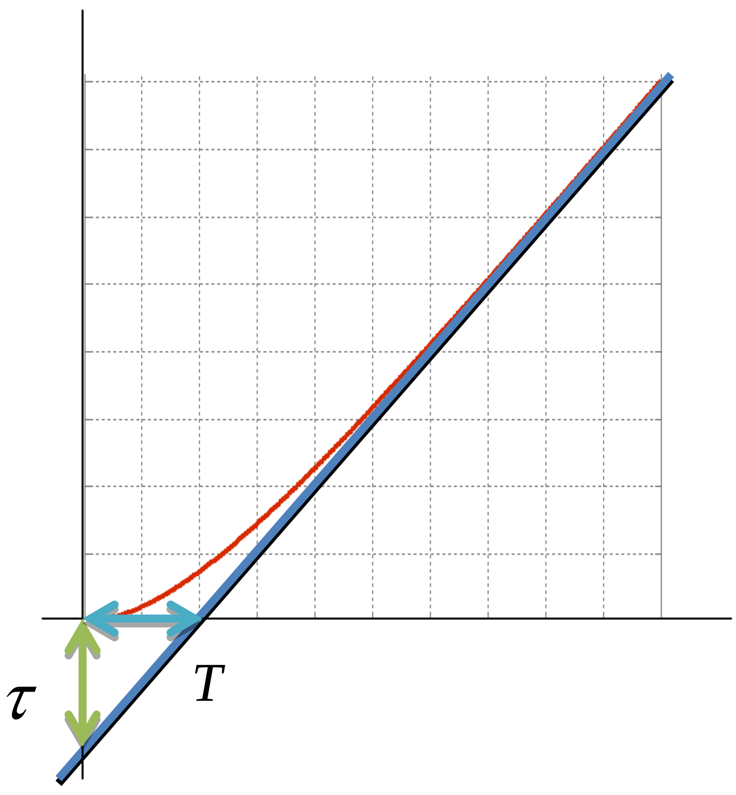


$$G(s) = \frac{k}{s(Ts+1)}$$

$$\frac{k}{s^2(Ts+1)} = \frac{k}{s^2} - \frac{Tk}{s} + \frac{Tk}{s+1/T}$$

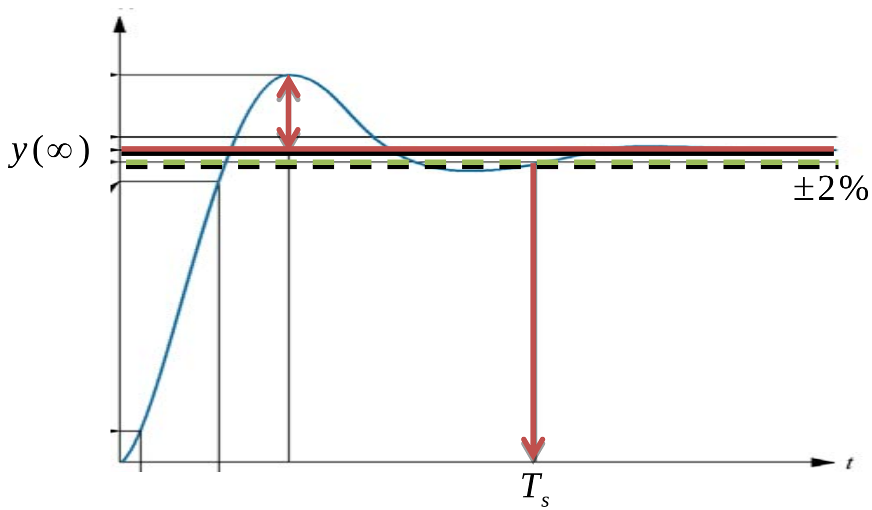
$$h(t) = kt - kT + kTe^{-t/T}$$

$$h_{\text{asymptota}}(t) = kt - kT = k(t-T)$$



Bump test

$$G(s) = k\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



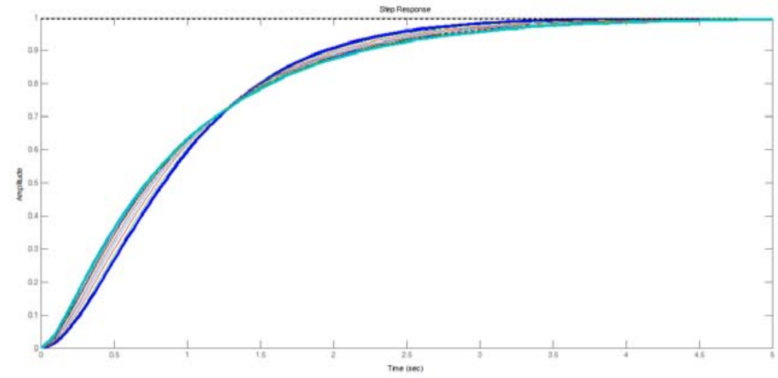
$$y(\infty), \%OS, T_s \quad \Rightarrow \quad \zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}, \quad \omega_n \doteq \frac{4}{\zeta T_s}, \quad k = \frac{y(\infty)}{u(\infty)}$$

Bump test

$$G(s)=\frac{k}{(1+T_1 s)(1+T_2 s)}e^{-sL},T_2\le T_1$$

$$y(t)=\begin{cases} k\left(1-\dfrac{T_1}{T_1-T_2}e^{-(t-L)/T_1}-\dfrac{T_2}{T_2-T_1}e^{-(t-L)/T_2}\right) & T_1\ne T_2 \\ k\left(1-e^{-(t-L)/T_1}-\dfrac{t}{T_1}e^{-(t-L)/T_1}\right) & T_1=T_2 \end{cases}$$

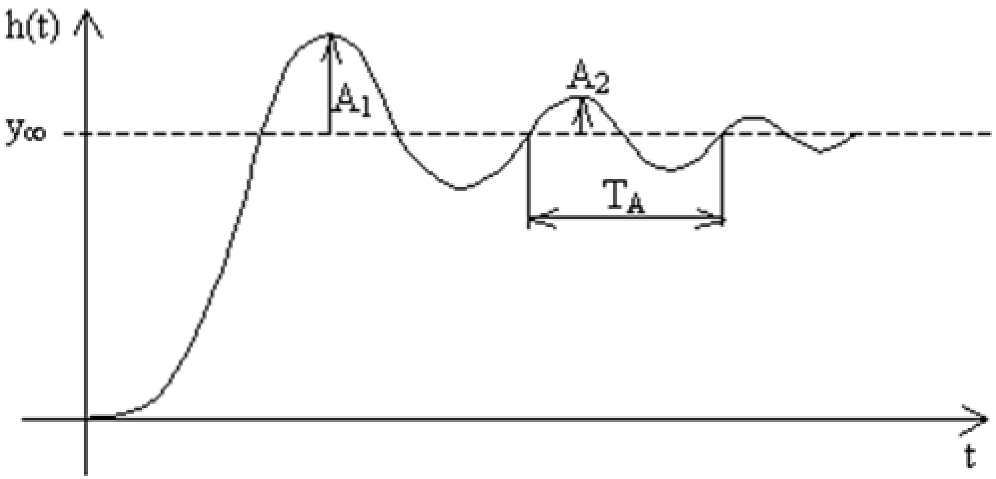
y



$$\tau=t/(T_1+T_2)$$

Strejc's method

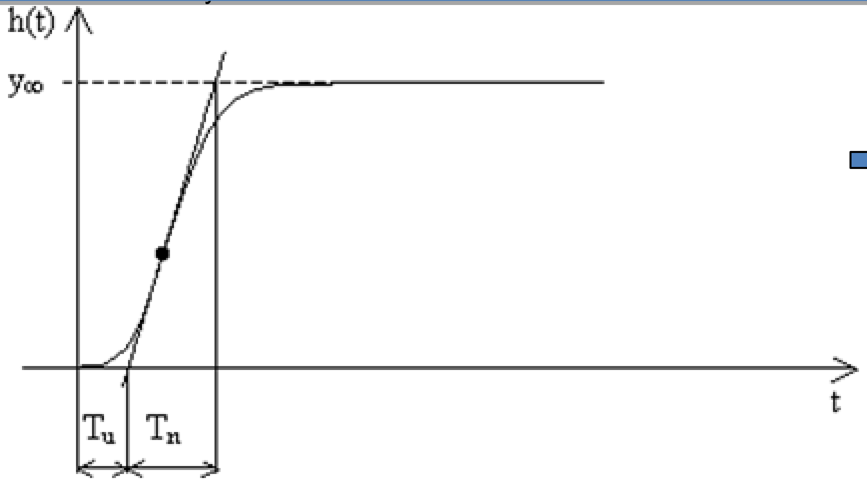
$$G(s)=\frac{k}{T_0^2 s^2+2\varsigma T_0+1}$$



$$y(\infty), A_1, A_2, T_A \Rightarrow k=\frac{y(\infty)}{u(\infty)}, \mu=\ln\frac{A_1}{A_2}, \zeta=\frac{\mu}{\sqrt{4\pi^2+\mu^2}}, T_0=\frac{1}{T_A}\sqrt{1-\zeta^2}$$

$$\left(\frac{A_1}{A_2}=e^{\sigma T_A}, \mu=\ln\frac{A_1}{A_2}=\sigma T_A=2\pi\frac{\sigma}{\omega_d}, \omega_d=\sqrt{\omega_n^2-\sigma^2}\right)$$

Strejc's method



$$\tau = \frac{T_u}{T_n}$$

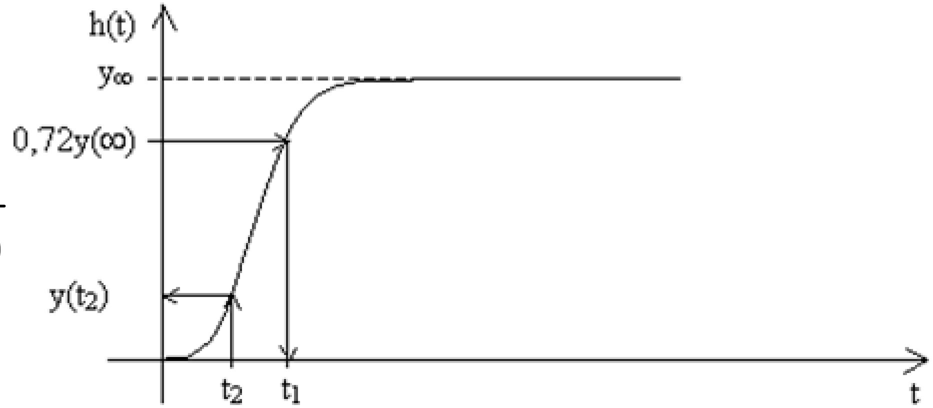
$$\tau < 0.1 \rightarrow G(s) = \frac{k}{(T_1 s + 1)(T_2 s + 1)}$$

$$\tau \geq 0.1 \rightarrow G(s) = \frac{k}{(Ts + 1)^n}$$

Strejc's method

$\tau < 0.1$

$$G(s) = \frac{k}{(T_1 s + 1)(T_2 s + 1)}$$



1) $k = \dfrac{y(\infty)}{u(\infty)}$

2) $t_1 : \quad y(t_1) = 0.72\, y(\infty)$

3) $T_1 + T_2 = \dfrac{t_1}{1.2564}$

4) $t_2 = 0,3574 \left(T_1 + T_2\right)$

5) $y(t_2)$ →

6) $\tau_2 = \dfrac{T_1}{T_2}$ ←

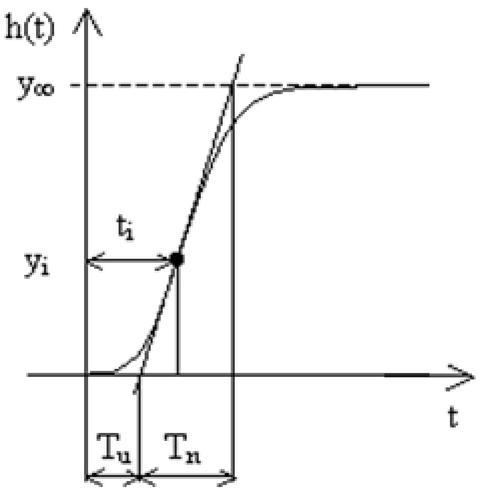
7) T_1, T_2

$y(t_2)$	τ_2	$y(t_2)$	τ_2
0,30	0,000	0,22	0,183
0,29	0,023	0,21	0,219
0,28	0,043	0,20	0,264
0,27	0,063	0,19	0,322
0,26	0,084	0,18	0,403
0,25	0,105	0,17	0,538
0,24	0,128	0,16	1,000
0,23	0,154		

Strejc's method

$\tau \geq 0.1$

$$G(s) = \frac{k}{(Ts+1)^n}$$



$$k = \frac{y(\infty)}{u(\infty)} \qquad y(\infty) = 1 \qquad \tau = \frac{T_u}{T_n}$$

$$y_i$$

n	2	3	4	5	6	7	8	9	10
τ	0,104	0,218	0,319	0,41	0,493	0,57	0,642	0,709	0,773
y_i	0,264	0,327	0,359	0,371	0,384	0,394	0,401	0,407	0,413

$$t_i : \; y(t_i) = y_i$$

$$T = \frac{t_i}{n-1} \qquad T = \frac{t_i - T_d}{n-1} \qquad T_d > 0$$