19 – Polynomial methods

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Automatic control

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Polynomials – properties. A review.

- Ring of polynomials (not field).
- Therefore: divisor, least common divisor, coprime polynolmials

$$g(s) = \operatorname{gcd}\left(a(s), b(s)\right) \Leftrightarrow \exists p(s), q(s), r(s), v(s):$$

$$a(s)p(s) + b(s)q(s) = g(s)$$
$$a(s)v(s) + b(s)w(s) = 0$$

$$U(s) = \begin{bmatrix} p(s) & q(s) \\ v(s) & w(s) \end{bmatrix}, \det U(s) \in \Re$$

- Division with remainder – the Euclid's algorithm:

$$a(s) = b(s)q(s) + r(s), \ \deg r(s) < \deg b(s)$$

- Linear equations $a(s)x(s) = b(s)$ do not have solutions typically since $x(s) = b(s)/a(s)$ is not generically a polynomial
- Diophantine equations:

$$\boxed{a(s)x(s) + b(s)y(s) = c(s)}$$

Properties of polynomial equations

Polynomial Diophantone equation

$$a(s)x(s)+b(s)y(s)=c(s)$$

$$g=\gcd(a,b)$$
$$\bar{a}=a/g$$
$$\bar{b}=b/g$$

- Solution exists iff $g\,|\,c$

All solutions:

$$x(s)=x'(s)-\bar{b}(s)t(s)$$
$$y(s)=y'(s)+\bar{a}(s)t(s)$$

$t(s)$ is a free polynomial parameter

Minimum degree solutions:

- There esists a unqiue solution such that $\deg x<\deg \bar{b}$
  i.e. minimal degree in x
- There esists a unqiue solution such that $\deg y<\deg \bar{a}$
  i.e. minimal degree in x
- These are the same if $\deg c<\deg a+\deg b$, otherwise thy differ

Pole placement

1. Select desirable CL poles. Compose the closed lool characteristic polynomial. Solve $a(s)x(s)+b(s)y(s)=c(s)$
2. Select appropriate solution.

Cese 1: $a(s), b(s)$ coprime

- no hidden modes
- $c(s)$ can be an arbitrary polynomial (of sufficient order / degree though – otherwise, non-proper controller y/x)

Case 2: $a(s), b(s)$ not coprime: $\gcd(a(s),b(s)) = g(s)$

- hidden modes

$$g(s)\bar{a}(s)x(s)+g(s)\bar{b}(s)y(s)=g(s)\bar{c}(s)$$

- $c(s)$ must contain $g(s)$
- hidden modes cannot be re-assigned (modified)
- other poles can be placed arbitrarily

Pole placement: proper controllers

In order to get a proper controller:

- if $G(s)$ is strictly proper, $\deg a(s) = n$,

then select

1) c(s) of degree at least $2n$-1and
2) select minimal-solution (minimal degree of y), tedy

- Tím je zaručeno, že vyjde ryzí regulátor řádu *n-1*

Explanation / “proof”:

n $\quad \le n$-1 $\quad \le n$-1 $\quad 2n$-1

$$a(s)x(s)+b(s)y(s)=c(s)$$

$= 2n$-2 $\quad \le 2n$-2

$$\deg x(s) = n-1$$

All stabilizing controllers

Assures closed-loop stability.

All stabilizing controllers for a given system G(s) = b(s)/a(s), with $a(s)$, $b(s)$, are given as C(s) = q(s)/p(s) where $p(s)$, $q(s)$ solve the equation

$$a(s)p(s)+b(s)q(s)=c(s)$$

for all stable polynomials $c(s)$

Conditions:

G(s) has no hiden unstable modes, i.e. $\gcd(a,b)$ must be stable

Proper controller

For proper system, select $\deg c(s) \geq 2\deg a(s) - 1$
and select minimum-degre-of $q(s)$ solution

Model matching

- make the closed loop transfer function equal to a prescribed one:

Exact model matching:

- Given the system $a(s), b(s)$ and requested model, i.e. $f(s), g(s)$
- Find controller, i.e. $p(s), q(s), r(s)$ so that
- resulting closed loop transfer function equals the model

Solution:

All such controllers fullfill

$$a(s)p(s)+b(s)q(s)=f(s)\bar{b}(s)t(s)$$
$$r(s)=\bar{g}(s)t(s)$$

- where $\bar{b}(s)/\bar{g}(s)=b(s)/g(s)$ coprime
- $t(s)$ is an arbitrary parameter