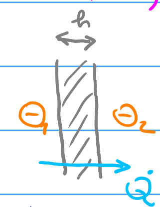
PROSTOROVĚ DISTRIBUOVANÉ SYSTÉMY
(SYSTÉMY S ROZPROSTŘENÝMI PARAMETRY)

(A3B35MSD 2014/15)

TEPELNÉ SYSTÉMY



$$\dot{Q} = \underbrace{H}_{\frac{A\cdot\lambda}{h}} \cdot (\Theta_1 - \Theta_2)$$

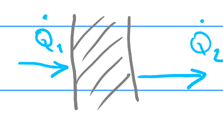
$h \to 0$:

$$\dot{Q} = -A\cdot\lambda\cdot \lim_{h\to 0} \frac{\Theta(x+h)-\Theta(x)}{h}$$

$$\frac{d}{dt} Q(x,t) = -\underbrace{A\cdot\lambda}_{k} \cdot \frac{\partial}{\partial x}\Theta(x,t)$$

$$\boxed{Q_t = -k\cdot\Theta_x}$$ ... Fourier

$$Q = m\cdot c\cdot\Theta = \int_V \varrho\cdot c\cdot\Theta\, dV$$



$$\dot{Q} = \frac{d}{dt}\int_a^b \varrho c\,\Theta\, dx = \boxed{\int_a^b \varrho c\,\dot{\Theta}\, dx} = \dot{Q}_1 - \dot{Q}_2$$

$$= -(\dot{Q}_2 - \dot{Q}_1)$$

$$= -(\dot{Q}(b) - \dot{Q}(a))$$

$$= \boxed{-\int_a^b \frac{\partial \dot{Q}}{\partial x}\, dx} + \int_a^b g(x,t)\, dx$$

$$\int_a^b \varrho c\,\dot{\Theta}\, dx = -\int_a^b \frac{\partial \dot{Q}}{\partial x}\, dx + \int_a^b g\, dx$$

$$\varrho c\,\dot{\Theta} = -\frac{\partial}{\partial x}\left(-k\frac{\partial}{\partial x}\Theta\right) + g$$

$$\boxed{\Theta_t - \frac{k}{\varrho c}\cdot\Theta_{xx} = \frac{1}{\varrho c}\cdot g}$$

Obecně pro 3d:

$$-\oint_A (\dot{Q}\cdot n)\, dA$$

$$= -\int_V (\nabla\cdot\dot{Q})\, dV$$

$$= -\int_V \nabla\cdot(-k\nabla\Theta)\, dV$$

$$\Downarrow$$

$$\int_V \varrho c\,\dot{\Theta}\, dV = \int_V k\underbrace{\nabla^2}_{\Delta}\Theta\, dV$$

$$\Rightarrow \boxed{\dot{\Theta} - \overbrace{\frac{k}{\varrho c}}^{\alpha}\cdot\Delta\Theta = \frac{1}{\varrho c}\, g}$$

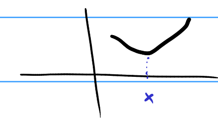
$$\boxed{\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}}$$

no heaters inside:

$$\boxed{\Theta_t = \alpha \cdot \Theta_{xx}} \quad \text{či} \quad \boxed{\dot{\Theta} = \alpha \cdot \Delta \Theta} \quad \text{... tepelná rovnice}$$

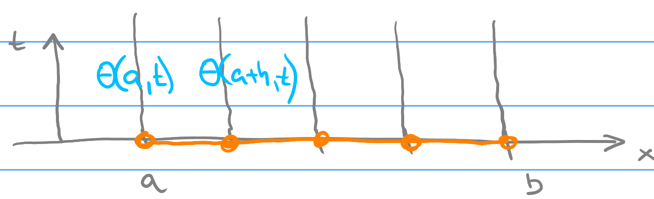
$\alpha > 0$

Θ_{xx} kvantifikuje zakřivení : $\Theta_{xx} > 0 \Rightarrow$



V ustáleném stavu ($t \to \infty$) : $\boxed{0 = \alpha \cdot \Theta_{xx}}$

Method of lines (MOL)



Místo $\Theta(x,t)$ použijeme

$$\begin{bmatrix} \Theta_1(t) \\ \Theta_2(t) \\ \vdots \\ \Theta_n(t) \end{bmatrix} \begin{matrix} \approx \Theta(a+h) \\ \approx \Theta(a+2h) \\ \vdots \\ \end{matrix} \quad \Bigg| \quad \begin{matrix} \Theta_0 := \Theta(a) \\ \Theta_{n+1} := \Theta(b) \end{matrix}$$

Aproximace $\Delta\Theta$ (či v 1D: $\frac{\partial^2 \Theta}{\partial x^2}$)

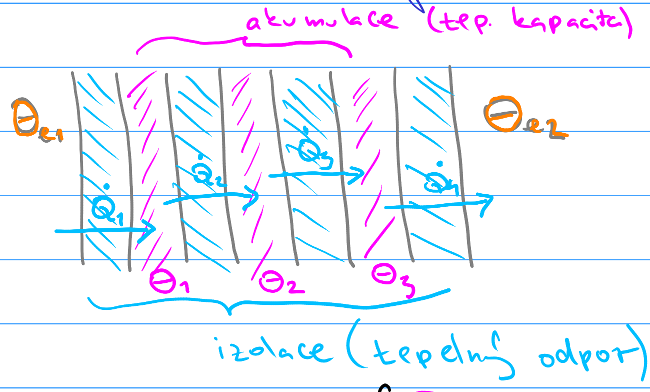
Centrální diference: $\frac{\partial \Theta}{\partial x} \approx \frac{\Theta(x + \frac{1}{2}h) - \Theta(x - \frac{1}{2}h)}{h}$

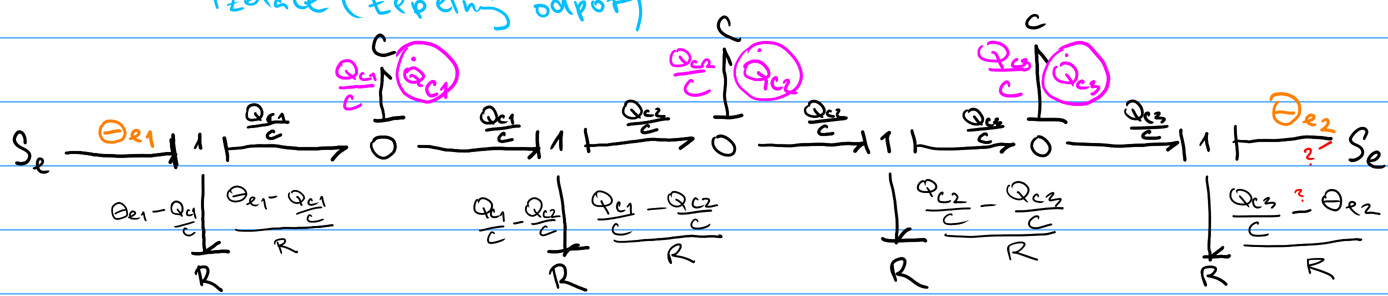
$$\frac{\partial^2 \Theta}{\partial x^2} \approx \frac{\frac{\Theta(x+h) - \Theta(x)}{h} - \frac{\Theta(x) - \Theta(x-h)}{h}}{h}$$

$$= \frac{1}{h^2}\left(\Theta(x+h) - 2\,\Theta(x) + \Theta(x-h)\right)$$

$$\begin{bmatrix} \dot{\Theta}_1 \\ \dot{\Theta}_2 \\ \vdots \\ \dot{\Theta}_n \end{bmatrix} = \frac{\alpha}{h^2} \cdot \begin{bmatrix} \Theta_2 - 2\Theta_1 + \Theta_0 \\ \Theta_3 - 2\Theta_2 + \Theta_1 \\ \vdots \\ \Theta_{n+1} - 2\Theta_n + \Theta_{n-1} \end{bmatrix} = \frac{\alpha}{h^2} \begin{bmatrix} -2 & 1 & \dots & \\ 1 & -2 & 1 & \dots \\ & \ddots & \ddots & \ddots \\ & & 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} \Theta_1 \\ \Theta_2 \\ \vdots \\ \Theta_n \end{bmatrix} + \frac{\alpha}{h^2} \begin{bmatrix} 1 & 0 \\ & \\ & \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \Theta_a \\ \Theta_b \end{bmatrix}$$

Diskretizace pomocí vazebních grafů





$$\dot{Q}_{c1} = \frac{\Theta_{e1} - \frac{Q_{c1}}{C}}{R} - \frac{\frac{Q_{c1}}{C} - \frac{Q_{c2}}{C}}{R} = \frac{\Theta_{e1}}{R} - \frac{2 \cdot Q_{c1}}{RC} + \frac{Q_{c2}}{RC}$$

$$\dot{Q}_{c2} = \frac{Q_{c1}}{RC} - \frac{2Q_{c2}}{RC} + \frac{Q_{c3}}{RC}$$

$$\dot{Q}_{c3} = \frac{Q_{c2}}{RC} - \frac{2\,Q_{c3}}{RC} \overset{?}{+} \frac{\Theta_{e2}}{R}$$

$Q_c = \Theta_c \cdot C$:

$$\cancel{C} \cdot \dot{\Theta}_{c1} = \frac{1}{R\cancel{C}} \Theta_{e1} - 2 \cdot \frac{\cancel{C} \cdot \Theta_{c1}}{RC} + \frac{\cancel{C} \cdot \Theta_{c2}}{RC}$$

$$\cancel{C} \cdot \dot{\Theta}_{c2} = \frac{\cancel{C} \cdot \Theta_{c1}}{RC} - 2\,\frac{\cancel{C}\,\Theta_{c2}}{RC} + \frac{\cancel{C} \cdot \Theta_{c3}}{RC}$$

$$\cancel{C} \cdot \dot{\Theta}_{c3} = \frac{\cancel{C} \cdot \Theta_{c2}}{RC} - 2 \cdot \frac{\cancel{C} \cdot \Theta_{c3}}{RC} \overset{?}{+} \frac{1}{R\cancel{C}} \cdot \Theta_{e2}$$

$$\begin{bmatrix} \dot{\Theta}_{c1} \\ \dot{\Theta}_{c2} \\ \dot{\Theta}_{c3} \end{bmatrix} = \frac{1}{RC} \begin{bmatrix} -2 & 1 & \cdot \\ -1 & -2 & 1 \\ \cdot & 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} \Theta_{c1} \\ \Theta_{c2} \\ \Theta_{c3} \end{bmatrix} + \begin{bmatrix} \frac{1}{RC} & 0 \\ 0 & 0 \\ 0 & \overset{?}{+}\frac{1}{RC} \end{bmatrix} \cdot \begin{bmatrix} \Theta_{e1} \\ \Theta_{e2} \end{bmatrix}$$

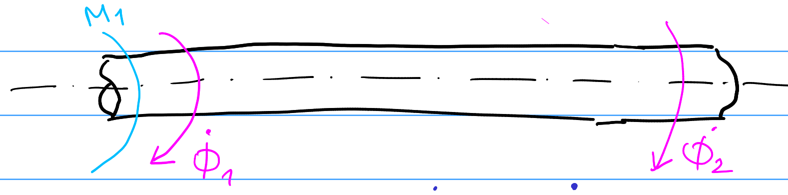
?? A ??

$$R = \frac{1}{H} = \frac{h}{A\cdot\lambda}$$

$$C = m\cdot c = A\cdot h\cdot\rho\cdot c$$

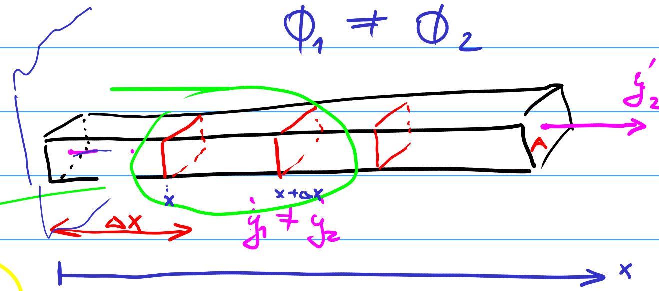
$$\boxed{\frac{1}{RC}} = \frac{\cancel{A}\cdot\lambda}{h\cdot\cancel{A}\cdot h\cdot\rho c} = \boxed{\underbrace{\frac{\lambda}{\rho\cdot c}}_{\alpha}\cdot\frac{1}{h^2}}$$

PRUŽNÁ HŘÍDEL



$$\dot\phi_1 \neq \dot\phi_2$$

nebo podélné kmitání:



$\sigma(x)$ $\sigma(x+\Delta x)$ Δx

STRESS

$$\underbrace{\Delta x\cdot A\cdot\rho}_{m}\cdot\ddot y(x,t) = A\cdot\sigma(x+\Delta x) - A\cdot\sigma(x)$$

směřuje ven $\quad / : \Delta x,\quad \Delta x \to 0$

$$A\rho\frac{\partial^2 y(x,t)}{\partial t^2} = \cancel{A}\cdot\frac{\partial\sigma(x,t)}{\partial x} \Rightarrow \frac{\partial^2 y}{\partial t^2} = \frac{1}{\rho}\frac{\partial\sigma}{\partial x}$$

STRAIN = RELATIVE DEFORMATION

$$\sigma(x,t) = \underset{\text{YOUNG}}{E}\cdot\frac{y(x)-y(x-h,t)}{\Delta x} \xRightarrow{\Delta x\to 0} \boxed{\sigma = E\cdot\frac{\partial y}{\partial x}}$$

$$\boxed{\frac{\partial^2 y}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 y}{\partial x^2}}$$

$$\boxed{\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}}$$ ... VLNOVÁ ROVNICE

c... rychlost šíření vlny

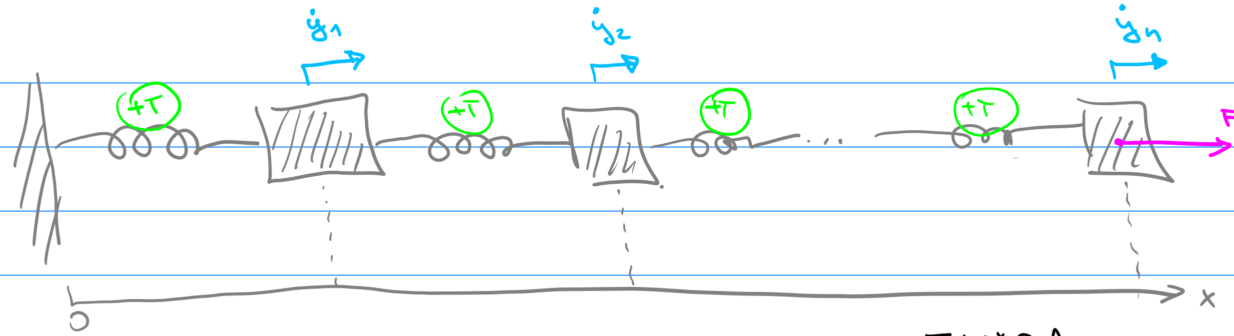
$m\cdot\dot y =: p \Rightarrow$ 1. rovnice: $\boxed{\dot p A = F(x+\Delta x) - F(x)} \Rightarrow \dot p_k = F_{k+1} - F_k$

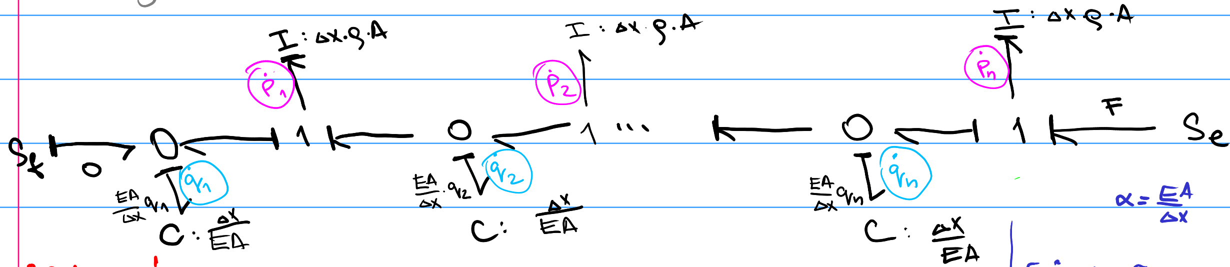
2. rovnice: $\boxed{F(x) = \frac{EA}{\Delta x}\cdot q(x)} \Rightarrow F_k = \frac{EA}{\Delta x}q_k$

$q := y(x) - y(x-h)$

$$\boxed{\dot p_k = \frac{EA}{\Delta x}\left(q_{k+1} - q_k\right)}$$

$$\boxed{\dot q_k = \dot y(x) - \dot y(x-h) = \frac{(p_k - p_{k-1})}{\Delta x\cdot\rho\cdot A}}$$





PRVNÍ A POSLEDNÍ:

$$\dot q_1 = \frac{1}{\Delta x\,\varrho\, A}\cdot p_1 \qquad\qquad \dot p_n = F - \frac{EA}{\Delta x} q_n$$

$$\alpha = \frac{EA}{\Delta x} \qquad \beta = \frac{1}{\Delta x\,\varrho\, A}$$

$$\boxed{\dot p_k = \frac{EA}{\Delta x}\left(q_{k+1} - q_k\right)} \qquad\qquad \boxed{\dot q_k = \frac{(p_k - p_{k-1})}{\Delta x\cdot\varrho\cdot A}}$$

$$\Downarrow\ \Delta x \to 0 \qquad\qquad \Downarrow\ \Delta x \to 0$$

$$\boxed{\frac{\partial p}{\partial t} = EA\,\frac{\partial q}{\partial x}} \qquad\qquad \boxed{\frac{\partial q}{\partial t} = \frac{1}{\varrho A}\cdot\frac{\partial p}{\partial x}}$$

$$\begin{bmatrix}\dot p_1\\ \dot p_2\\ \vdots\\ \dot p_n\\ \dot q_1\\ \dot q_2\\ \vdots\\ \dot q_n\end{bmatrix} = \begin{bmatrix} & & -\alpha & \alpha & 0 & 0\\ & & & -\alpha & \alpha & 0\\ & & & & \ddots & \\ & & & & & -\alpha\\ \beta & & & & & \\ -\beta & \beta & & & & \\ & \ddots & \beta & & & \\ & & -\beta & \beta & & \end{bmatrix} \begin{bmatrix} p_1\\ p_2\\ \vdots\\ p_n\\ q_1\\ q_2\\ \vdots\\ q_n\end{bmatrix} + \begin{bmatrix} \\ \\ 1 \\ \\ \\ \end{bmatrix}\cdot F$$

$$\boxed{\begin{bmatrix}\frac{\partial p}{\partial t}\\ \frac{\partial q}{\partial t}\end{bmatrix} = \begin{bmatrix} 0 & EA\frac{\partial}{\partial x}\\ \frac{1}{\varrho A}\frac{\partial}{\partial x} & 0\end{bmatrix}\cdot\begin{bmatrix} p\\ q\end{bmatrix}}$$

$$\begin{bmatrix} y_1\\ y_2\\ \vdots\\ y_n\end{bmatrix} = \begin{bmatrix} 1 & & & & \\ 1 & 1 & & & \\ 1 & 1 & 1 & & \\ \vdots & & & \ddots & \\ 1 & 1 & 1 & 1 & 1\end{bmatrix}\cdot\begin{bmatrix} q_1\\ q_2\\ \vdots\\ q_n\end{bmatrix}$$

$q_1 = y_1$

$q_2 = y_2 - y_1$

$q_3 = y_3 - y_2$

$\vdots$

$q_k = y_k - y_{k-1}$

$q_n = y_n - y_{n-1}$

ALE TAKÉ:

$$v = \dot y$$

$$\Downarrow$$

$$\boxed{\begin{bmatrix}\frac{\partial y}{\partial t}\\ \frac{\partial v}{\partial t}\end{bmatrix} = \begin{bmatrix} 0 & 1\\ \frac{E}{\varrho}\frac{\partial^2}{\partial x^2} & 0\end{bmatrix}\cdot\begin{bmatrix} y\\ v\end{bmatrix}}$$

! VŽDY UPŘESNIT OKRAJOVÉ A POČÁTEČNÍ PODMÍNKY